1. Determine whether the following functions are *piecewise continuous* (pwc) on the interval $[0, 1]$.

$$a)\ f(x) = \ln(x), \qquad b)\ f(x) = x^{\frac{1}{3}}, \qquad c)\ f(x) = (1-x)^{-1},$$

$$d)\ f(x) = \begin{cases} 1 & x < \frac{1}{4} \\ 2 & \frac{1}{4} < x < \frac{1}{2} \\ 0 & x > \frac{1}{2} \end{cases}.$$

2. Show that the set of functions $\left\{\cos(\frac{n\pi x}{L})\right\}$, $n = 1, 2, \ldots$, is orthogonal on the interval $[-L, L]$ with respect to the standard inner-product.
Hint: $2\cos A \cos B = \cos(A - B) + \cos(A + B)$.

3. Find the corresponding set of *orthonormal* functions for the orthogonal set given in question 2.

4. Show that the set of functions,

$$\left\{1,\, \cos\frac{n\pi x}{L},\, \sin\frac{m\pi x}{L}\right\}, \quad n, m = 1, 2, \ldots$$

is an orthogonal set on the interval $[-L, L]$.

5. Define a *weighted* inner-product of two functions on the interval $[-1, 1]$ via

$$(f(x), g(x)) = \int_{-1}^{1} \frac{1}{\sqrt{1-x^2}} f(x)g(x)\, dx.$$

Show that the functions 1 and x are orthogonal with respect to this inner-product.

6. Determine whether the following functions are *piecewise smooth* (pws) on the interval $[0, 1]$. Give reasons for your answers.

$$a)\ f(x) = \ln(x), \qquad b)\ f(x) = x^{\frac{1}{3}}, \qquad c)\ f(x) = x^{-1},$$

$$d)\ f(x) = \begin{cases} 1 & x < \frac{1}{4} \\ 2 & \frac{1}{4} < x < \frac{1}{2} \\ 0 & x > \frac{1}{2} \end{cases}.$$

7. Sketch the periodic extensions of the following functions, defined on the interval $[-L, L]$, to the interval $[-3L, 3L]$.

$$a)\ f(x) = x \qquad b)\ f(x) = x^2 \qquad c)\ f(x) = \begin{cases} 1 & x \leq 0 \\ 2 & x > 0 \end{cases}.$$