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1. Find a unit vector $\hat{\mathbf{n}}$ that is normal to each of the following surfaces:

$$a)\, z = 2 - x - y, \quad b)\, z = \left(1 - x^2\right)^{\frac{1}{2}}, \quad c)\ \left(1 - x^2 - y^2\right)^{\frac{1}{2}}.$$

2. Find a unit vector that is normal to the upper hemisphere of the unit sphere and a unit vector that is normal to the lower hemisphere of the unit sphere.

3. Evaluate the following surface integrals: $\int\int_S G(x, y, z)\, dS$, where

   a) $G(x, y, z) = z$, S is the portion of the plane $x + y + z = 1$ in the first octant (i.e. $x \geq 0$, $y \geq 0$, $z \geq 0$);

   b) $G(x, y, z) = \frac{1}{1+4(x^2+y^2)}$, S is the portion of the paraboloid $z = x^2 + y^2$ between $z = 0$ and $z = 1$.

4. Without resorting to the Divergence Theorem, evaluate each of the following surface integrals of the form: $\int\int_S \mathbf{F} \cdot \mathbf{n}\, dS$ where $\mathbf{n}$ is a unit normal vector to the surface in question:

   a) $\mathbf{F} = x\mathbf{i} - z\mathbf{k}$, S is the portion of the plane $x + y + 2z = 2$ in the first octant.

   b) $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{z}$ where S is the surface of the sphere $z^2 + x^2 + y^2 = a^2$.

5. Verify the divergence theorem for the following cases:

   a) $\mathbf{F} = (y - x)\mathbf{i} + (y - z)\mathbf{j} + (x - y)\mathbf{k}$, V is the unit cube.

   b) $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, V is the sphere of radius a.

6. Verify Stokes' theorem in the following cases:

   a) $\mathbf{F} = \left(y + y^2\right)\mathbf{k}$, C is the perimeter of the triangle, S, with vertices $(0, 0, 1)$, $(1, 0, 0)$ and $(0, 1, 0)$.

   b) $\mathbf{F} = -y\mathbf{i} + x\mathbf{j}$, S is the hemisphere $x^2 + y^2 + z^2 = 4$, $z \geq 0$, and C is the closed curve $x^2 + y^2 = 4$ in the x-y plane.

7. Consider a box described by the intersection of the six planes

$$x = 0, \quad x = 1, \quad y = 0, \quad y = 2, \quad z = 0, \quad z = 1.$$

   Suppose we remove the face lying in the $z = 0$ plane (so the box is open at the bottom). Let S denote the surface of the open box and let C denote the closed curve lying in the plane $z = 0$ that is the boundary of the open side of the box. Verifty Stokes' theorem in the case where:

$$\mathbf{F} = y\mathbf{i} - x^2 z\mathbf{j} + xy\mathbf{k}.$$

   HINT: Sketch the open box and label the five faces $S_1, \ldots, S_5$. Similarly, label the four line segments $C_1, \cdots, C_4$ of the boundary curve corresponding to the open side.