Suggested reading: Review your lecture notes on partial differentiation and polar coordinates from first year calculus courses.

Cartesian, Cylindrical and Spherical Co-ordinates

1. Convert the following sets of cylindrical coordinates to Cartesian coordinates

$$a)\ \left(2, \frac{2\pi}{3}, 2\right) \quad b)\ \left(\pi, \frac{\pi}{2}, 1\right) \quad c)\ (3, 0, -6)$$

2. Convert the following sets of spherical coordinates to Cartesian coordinates;

$$a)\ (3, 0, \pi) \quad b)\ \left(2, \frac{\pi}{4}, \frac{\pi}{3}\right) \quad c)\ \left(3, 0, \frac{2\pi}{3}\right)$$

3. Convert the following sets of Cartesian coordinates to i) cylindrical and ii) spherical coordinates:

$$a)\ (-1, 0, 2) \quad b)\ \left(-1, \sqrt{3}, 13\right) \quad c)\ (0, 2, -1)$$

4. Describe how the following equations can be interpreted as surfaces (in the relevant co-ordinate systems) and sketch them.

cartesian coordinates:	$a)\ x^2 + y^2 + z^2 = 2$	$b)\ z = 4 + x + y$	$c)\ z = x^2 + y^2$
cylindrical coordinates:	$d)\ r\cos\theta = 3$	$e)\ \theta = \frac{\pi}{3}$	$f)\ r^2 = 4$
spherical coordinates:	$g)\ \rho = \frac{\pi}{4}$	$h)\ \phi = \frac{\pi}{4}$	$i)\ \rho\sin\phi = 2.$

5. Which points in $\mathbb{R}^3$ have the same coordinates in the Cartesian and cylindrical systems?

Partial differentiation

6. (Revision) Find the first partial derivatives of the following functions $z = z(x, y)$,

$$a)\ z = y\ln x, \quad b)\ z = e^{x^2-y^2}, \quad c)\ z = \sin(x+y)\cos(2x-y).$$

7. (Revision) Determine which of the following functions is a solution of Laplace's equation in two space dimensions i.e. $u_{xx} + u_{yy} = 0$.

$$a)\ x^2 + y^2 \quad b)\ \ln\sqrt{x^2+y^2} \quad c)\ e^{-x}\cos y - e^{-y}\sin x$$

8. (Revision) Determine which of the following functions is a solution of the one-dimensional wave equation $u_{tt} = u_{xx}$

$$a)\ \sin(x-t) + \ln(x+t) \quad b)\ (x-t)^{-2} + 2\cos(x+t) \quad c)\ \exp(x+t) + \exp(x-t)$$

Can you spot what all these functions have in common?

9. Suppose we want to express a function $u = u(r, \theta, z)$ in cylindrical coordinates. Derive formulae for the derivatives u_x, u_y, u_z (where x, y, z are the standard Cartesian coordinates) in terms of the derivatives u_r and u_θ and u_z. Hence convert Laplace's equation in Cartesian coordinates:

$$u_{xx} + u_{yy} + u_{zz} = 0,$$

into cylindrical coordinates.

Hint: Use the chain rule for partial derivatives.