MATH36001 Mid-Term Test: Information 2015

The mid-term test will take place during the Wednesday lecture on **on 11th November.** The rooms are **Stopford Lecture Theatre 2 and 6**. (Refer to the course webpage for info!)
The rubric is to answer all the questions, which are worth a total of 20 marks.
The test is on all the material discussed in the lectures and tutorials up to date.
Below is a sample test.

Total 20 marks. Answer all 8 questions.

1. True or false: $p(A)A = Ap(A)$ for any $A \in \mathbb{C}^{n\times n}$ and any polynomial p? Justify your answer with a proof or a counterexample. [3 marks]

2. For $A \in \mathbb{C}^{n\times n}$, give two conditions equivalent to A being singular. [2 marks]

3. For nonzero $u, v \in \mathbb{C}^n$ let $A = I - uv^*$. Show that $\mu = 1 - v^*u$ is an eigenvalue of A with associated eigenvector u. What are the other eigenvalues of A? For which value of μ is A is idempotent (i.e., $A^2 = A$)? [4 marks]

4. State the spectral theorem. [2 marks]

5. True or false: *An $n \times n$ matrix has a set of n linearly independent eigenvectors.* Justify your answer with a proof or a counterexample.

[1 marks]

6. Ascertain if the following matrices are unitarily similar to a diagonal matrix:

- A such that $A^*A = AA^*$.
- B has n orthogonal eigenvectors.
- C has n distinct eigenvalues.

[3 marks]

7. Suppose the Jordan form of A is

$$J = \mathrm{diag}\left(\begin{bmatrix} 4 & 1 & \\ & 4 & 1 \\ & & 4 \end{bmatrix}, \begin{bmatrix} 4 & 1 \\ & 4 \end{bmatrix}, \begin{bmatrix} 3 & 1 \\ & 3 \end{bmatrix}, [\,2\,], [\,2\,] \right).$$

(a) Determine the algebraic and geometric multiplicity of each distinct eigenvalue of A.

(b) Give the linearly independent eigenvectors of the Jordan matrix J.

(c) Determine the minimal polynomial of A.

[3 marks]

8. Define the vector p-norm and state the Hölder inequality. [2 marks]