MATH10131

Two and a half hours

THE UNIVERSITY OF MANCHESTER

CALCULUS AND VECTORS B

XX January 2019

X.00 – X.30

Answer **ALL FIFTEEN** questions.

The use of electronic calculators and formula tables are not permitted.

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1. Sketch in the complex plane the region where complex values of z satisfy

$$3 <| 3z - 9 + i |< 6.$$

[2 marks]

2. Sketch graphs of the following real-valued functions of x satisfying

(a)

$$f(x) = -e^{-|x+2|} + 1;$$

(b)

$$f(x) = \ln(4x^2).$$

[4 marks]

3. A function f is defined by

$$f(x) = -\sqrt{-x-1}.$$

(a) Find a formula for the inverse function $f^{-1}(x)$.

(b) Sketch the graphs of $f^{-1}(x)$ and $f(x)$ using the same coordinate axes.

[4 marks]

4. By using implicit differentiation, find the derivative of the inverse trigonometric function

$$\cos^{-1}(x).$$

[4 marks]

5. Find the limits

(a)

$$\lim_{x\to 0} \frac{1-\cos^2(x)}{x^2};$$

(b)

$$\lim_{x\to\infty} \sqrt{x^2+10x} - x.$$

[4 marks]

6. Find an equation of the tangent line to the elipse

$$x^2 + xy + y^2 = 3$$

at the point $(1, 1)$.

[4 marks]

7. Sketch the region between the curves $y^2 = x$ and $x^2 = y$ in Cartesian coordinates (x, y) and find the area of this region.

[4 marks]

8. Find the first three nonzero terms in the Maclaurian series for the function

$$f(x) = xe^{-x^2}.$$

[4 marks]

9. The distance ℓ of the point $\vec{P}$ from the plane $(\vec{r} - \vec{r}_0) \cdot \vec{n} = 0$ can be calculated as

$$\ell = \frac{|(\vec{P} - \vec{r}_0) \cdot \vec{n}|}{|\vec{n}|}.$$

Find the distance between the point $(2, 0, 1)$ and the plane that passes through the three points $(2, 3, 4)$, $(1, 2, 4)$, $(4, 2, 1)$.

[6 marks]

10. The distance ℓ of the point $\vec{P}$ from the line $\vec{r} = \vec{r}_0 + t\,\vec{v}$ can be calculated as

$$\ell = \frac{|(\vec{P} - \vec{r}_0) \times \vec{v}|}{|\vec{v}|}.$$

Find the distance between the point $(2, 2, 1)$ and the line

$$2x - 4 = y - 3 = -\frac{z}{2}.$$

[6 marks]

11. Sketch the region D between $y = 2x$, $y = 2$, $x = 0$ and evaluate the following integral

$$\int\int_D e^{y^2} dy dx.$$

[8 marks]

12. Use polar coordinates to evaluate the following integral

$$\int\int_D dA,$$

where D is the interior of the curve $x^2 - 6x + y^2 = 0$.

[8 marks]

13. Find the volume of the solid bounded by the plane $z = 0$ and the paraboloid

$$z + x^2 + y^2 = 1.$$

[5 marks]

14. Find an equation of the tangent plane to the surface

$$z = \sqrt{x + e^{4y}}$$

at the point $(3, 0, 2)$.

[6 marks]

15. Find all critical points of the function

$$f(x, y) = e^{4y - x^2 - y^2}$$

and identify whether each one is a maximum, a minimum or a saddle point.

[6 marks]

END OF EXAMINATION PAPER