20912 Introduction to Financial Mathematics

TEST - 10 March 2008 (9.00-9.50)

Surname___________________________**Name:**_____________________________

(please write in BLOCK CAPITALS)

1. The stock price obeys the stochastic differential equation $dS = \mu S dt + \sigma S dW$.

(a) (2 marks) Suppose that the expected return from a stock is 16% per annum and the volatility is 45% per annum. Initial stock price is \$80. By using $dW \approx X(\Delta t)^{\frac{1}{2}}$, where X is $N(0,1)$, calculate the increase ΔS in the stock price during four days.

Solution:

(b) (3 marks) By using Ito's Lemma $df = \left(\frac{\partial f}{\partial t} + \mu S \frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2}\right) dt + \sigma S \frac{\partial f}{\partial S} dW$, find the SDE satisfied by $f(t,S) = AtS^2 \quad (A - constant)$

Solution:

2. Using the put-call parity formula $P - C + S = E\exp(-rT)$

(a) (2 marks) Find a lower bound for the European call option with exercise price \$35 when the stock price is \$40, the time to maturity is six months, and the risk-free rate of interest is 5% p.a.;

Solution:

(b) (3 marks) Consider the situation where the European call option is \$5 which is less than the theoretical minimum. Show that there exists an arbitrage opportunity.

Solution:

3. (4 marks) Draw the payoff diagram of the portfolio: long one share, short one call and short two puts, all with strike price E.

Payoff diagram:

4. (6 marks) Draw the payoff diagram of the portfolio: short 2 shares, long three puts with strike price E_1 and long four calls with strike price E_2 ($E_1 < E_2$).

Payoff diagram: