Calculus and Vectors B - MATH10131

Problem Sheet for Week 8 — Integration & Vectors

***Suggested reading*:** 'Stewart' Chapters 6, 8 and 12

Easy Questions

1. For each of the following pairs of vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ and scalar c evaluate all of the quantities

$$|\vec{\mathbf{a}}|, \quad |\vec{\mathbf{b}}|, \quad \vec{\mathbf{a}}+\vec{\mathbf{b}}, \quad \vec{\mathbf{a}}-\vec{\mathbf{b}}, \quad c\,\vec{\mathbf{a}}, \quad c\,\vec{\mathbf{b}} \quad \text{and} \quad \vec{\mathbf{a}}\cdot\vec{\mathbf{b}}$$

   and state whether or not $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are orthogonal

   (a) $\vec{\mathbf{a}} = (1, -\sqrt{2})$, $\vec{\mathbf{b}} = (\sqrt{3}, -1)$ and $c = \sqrt{2}$

   (b) $\vec{\mathbf{a}} = (-5, 1, 2)$, $\vec{\mathbf{b}} = (1, 3, 1)$ and $c = -\frac{1}{3}$

2. Find the x and y coordinates of the vectors described below, and write the vector in the form $x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}}$ (*Note*: conventionally, angles are always taken to be rotated anti-clockwise)

   (a) a vector of length $\sqrt{2}$ at an angle of $\frac{3}{4}\pi$ from the x-axis

   (b)★ a vector of length 2 at an angle of $\frac{3}{2}\pi$ from the x-axis

   (c) a vector of length $\sqrt{2}/3$ pointing midway between the directions of the x-axis and the y-axis

   Sketch all of these vectors on a single set of axes.

Standard Questions

3. Sketch the regions described in Cartesian coordinates (x, y) by each of the following examples and find the area of each region.

   ★(a) the region between the curves $y = x^2$ and $y = x^4$

   (b) the region between the curves $x^2 = 2py$ and $y^2 = 2px$

   ★(c) the region between $y = \sin x$, $y = e^x$, $x = 0$ and $x = \frac{1}{2}\pi$

   (d) the region between $y = \ln x$, $y = 1$, $y = 0$ and $x - y^2 + 1 = 0$

4. Sketch the regions described in Polar coordinates (r, θ) by each of the following examples and find the area of each region.

   ★(a) the region surrounded by the 'cardioid' $r = a(1 + cos\theta)$ for $\theta \in [0, 2\pi]$

   (b) the region between the radial line $\theta = 0$ and the spiral $r = 2\theta$ for $\theta \in [0, 2\pi]$

5. Sketch the the straight line $y = 4 - x$ and find its length from $x = 0$ to $x = 2$
