Calculus and Vectors B - MATH10131

Problem Sheet for Week 7 — More Integration

***Suggested reading*:** 'Stewart' Chapters 5, 6 and 7 (and chapters studied earlier)

Easy Questions

1. Evaluate the following indefinite integrals (you should be able to write down the answers immediately)

   (a) $\displaystyle\int \frac{14x^6 - 3x^2}{2x^7 - x^3 + 1}\,\mathrm{d}x$ (b) $\displaystyle\int \frac{\sec^2\theta}{\tan\theta}\,\mathrm{d}\theta$ (c) $\displaystyle\int \frac{1/r}{\ln|r|}\,\mathrm{d}r$

   (d) $\displaystyle\int \frac{\sin\theta}{\cos\theta}\,\mathrm{d}\theta$ (e) $\displaystyle\int \frac{\cos\theta}{\sin\theta}\,\mathrm{d}\theta$ (f) $\displaystyle\int \frac{\sinh y}{\cosh y}\,\mathrm{d}y$

Standard Questions

2. Confirm (by differentiating) that the product $\sin x\cos x$ can be integrated in different ways to give the three results

   $$\int \sin x\cos x\,\mathrm{d}x = \tfrac{1}{2}\sin^2 x + C, \quad \int \sin x\cos x\,\mathrm{d}x = -\tfrac{1}{2}\cos^2 x + C, \quad \int \sin x\cos x\,\mathrm{d}x = -\tfrac{1}{4}\cos(2x) + C$$

   Is it true that $\frac{1}{2}\sin^2 x = -\frac{1}{2}\cos^2 x = -\frac{1}{4}\cos(2x)$?

   If not, why is it that the integral can be written in any of these three ways?

3. Evaluate the indefinite integrals

   (★a) $\displaystyle\int x^2\ln(1+x^3)\,\mathrm{d}x$ (b) $\displaystyle\int t\sin(t^2)\,e^{1+2\cos t^2}\,\mathrm{d}t$ (c) $\displaystyle\int \frac{\cos\theta\sin\theta}{2\cos\theta - 5}\,\mathrm{d}\theta$

   (d) $\displaystyle\int \frac{5-x}{\sqrt{2x-3}}\,\mathrm{d}x$ (e) $\displaystyle\int \cosh(r^3 - 3r)\,(r^2-1)\,\mathrm{d}r$ (f) $\displaystyle\int z\sqrt{4-z^2}\,\mathrm{d}z$

4. Evaluate the definite integrals

   ((a) $\displaystyle\int_0^{\frac{1}{2}\sqrt{\pi}} \big(x - \tfrac{1}{2}\sqrt{\pi}\,\big)\sin(\tfrac{1}{2}\pi + x\sqrt{\pi} - x^2)\,\mathrm{d}x$ (b) $\displaystyle\int_{-2}^{5/2} \frac{t+1}{\sqrt{5-2t}}\,\mathrm{d}t$

   (★c) $\displaystyle\int_0^{\pi/2} \frac{\sin^2\theta\cos\theta}{2-\sin\theta}\,\mathrm{d}\theta$ (d) $\displaystyle\int_0^3 \frac{4s}{\sqrt[3]{9-s^2}}\,\mathrm{d}s$

5. Evaluate the indefinite integrals

   (★a) $\displaystyle\int \frac{3x^2 - 6x + 6}{(1-2x)(x-1)(x+2)}\,\mathrm{d}x$