

Calculus and Vectors B - MATH10131

Problem Sheet for Week 2

Functions and Graphs

Suggested reading: ‘Stewart’ chapter 1

Note: This problem sheet (for week 2) introduces some basic concepts about functions.

Easy Questions

- What are the domains of each of the following functions?
(a) $\sqrt{x+4}$ (b) $\frac{t}{t-1}$ (c) $\sqrt{v} + \sqrt[3]{1-v}$ (d) $\frac{1}{\sqrt[4]{p(2-p)}}$
- Sketch the graphs of the following functions. Explain very briefly how you got the graphs.
(a) $\sqrt{x^2}$ (b) $\sqrt{t-4}$ (c) $|3s+1|$ (d) $|x^3+1|$
- Are the following true or false? Give a corrected version of those that are false.
(a) \tan has range \mathbb{R} (b) the range of \cos is \mathbb{R} (c) \sec has domain \mathbb{R}
- *4. Sketch the graphs of the following functions (do not use a calculator).
(a) $4^x - 3$ (b) -2^{-x} (c) $3 - e^x$ (d) $\ln(5-x) - 3$
5. Find the exact value of each expression (do not use a calculator).
(a) $\sin^{-1}(\sqrt{3}/2)$ (b) $\tan^{-1}\sqrt{3}$ (c) $\sin(\sin^{-1}0.542)$ (d) $\tan^{-1}(\tan \frac{4\pi}{3})$
- *6. Find a formula for the inverse of each of the functions (be sure to identify the domain of each inverse)
(a) $\sqrt{9-3x}$ (b) $\exp(x^3)$ (c) $\ln(x+3)$
7. Add, subtract, multiply and divide the functions $\sqrt{x(2-x)}$ and $1-x^2$.
In each case, what is the domain?

Standard Questions

8. Sketch the graph of the relation $|x| + |y| = 1$. Does this relation represent a function?
9. Sketch the graphs of the following functions. Explain very briefly how you got each graph.
(a) $\frac{1}{1+t^2}$ (b) $\frac{3x+|x|}{x}$ (c) $\sqrt{\frac{x}{x-1}}$
10. Sketch the graphs of the following functions. Explain very briefly how you got each graph.
(a) $f(x) = \begin{cases} x+2 & \text{if } x \leq -1 \\ x^2 & \text{if } x > -1 \end{cases}$ (b) $g(t) = \begin{cases} \sqrt{t-1} & \text{if } t \geq 1 \\ -\sqrt{1-t} & \text{if } t < 1 \end{cases}$
11. For the function $f(x) = 1 - 2/x^2$, with domain $x > 0$, find an explicit formula for the inverse function f^{-1} . Sketch the curves of $y = f(x)$, $y = f^{-1}(x)$ and $y = x$, all on the same graph.
- *12. A function f is defined so that $f(x) = \sqrt{3 - e^{2x}}$
 - what is the domain of f
 - find a formula for the inverse function f^{-1}
 - what is the domain of f^{-1}

Harder Questions

13. (Scary problem!) If a bacteria population in a body starts with 100 bacteria and doubles every three hours
 - what is the number of bacteria $n(t)$ after t hours? (hint: use the exponential function)
 - find the inverse of this function and explain its meaning;
 - when will the population reach 50000? (it's over)
14. Sketch the graphs of $|x|^a + |y|^a = 1$, where a is a constant; consider all possible values of a from the interval $[1, \infty)$.