Problem Sheet 11 ***MAHT10131*** **Tangent Planes, Double Integrals**

***Suggested reading*:** 'Stewart' Chapters 14,15

Easy Questions

★1. Find an equation of the tangent plane to the given surface at the given point:

(a) $z = 4x^2 - y^2 + 2y, \quad (-1, 2, 4)$. Ans: $z = -8x - 2y$

(b) $z = y\cos(x - y), \quad (2, 2, 2)$.

2. The double integral $\iint_R \mathrm{d}x\,\mathrm{d}y$, which can also be written as $\iint_R 1\,\mathrm{d}x\,\mathrm{d}y$, gives the area of the region R.
Use double integrals to find the area of each of the following regions:

(a) the rectangle with vertices at $(0, 0)$, $(2, 0)$, $(2, 1)$ and $(0, 1)$.

(b) the triangle with sides formed by the x-axis and the straight lines $x = 1$ and $y = x$.

(c) the triangle between the intersections of the y-axis and the straight lines $y = x$ and $y = 1$.

(d) the region where $0 \leq y \leq \sin x$ over the interval $0 \leq x \leq \pi$.

(e) the finite region between the curve $y = e^x$ and the straight lines $y = 1$ and $x = 2$.

Standard Questions

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3. Find the directional derivative of the function at the given point in the direction of the vector $\vec{\mathbf{v}}$:

(a)★ $f(x, y) = 1 + 2x\sqrt{y}, \quad (3, 4), \quad \vec{\mathbf{v}} = (4, -3)$.

(b) $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}, \quad (1, 2, -2), \quad \vec{\mathbf{v}} = (-6, 6, -3)$.

4. Sketch each region of integration and evaluate each of the following double integrals:

(a) $\iint_R y\,\mathrm{d}x\,\mathrm{d}y$ where R is the region between the parabola $y = (x - 2)^2$ and the line $y = x$.

★(b) $\iint_R 2x\,\mathrm{d}x\,\mathrm{d}y$ where R is the region between the parabolæ $x = 2(y^2 - 1)$ and $x = \frac{1}{2}(1 - y^2)$.
Ans: -4

(c) $\iint_R x\,\mathrm{d}x\,\mathrm{d}y$ where R is the region between the parabola $y = (x - 2)^2$ and the line $y = 4 - x$.

(d) $\iint_R \frac{x}{1 + y^5}\,\mathrm{d}x\,\mathrm{d}y$ where R is the region bounded by the y-axis, the curve $y = \sqrt{x}$ and the line $y = 2$.
Ans: $\frac{1}{10}\ln 33$

(e) $\iint_R y\cos x^5\,\mathrm{d}x\,\mathrm{d}y$ where R is the region bounded by the x-axis, the parabola $y = x^2$ and the line $x = 2$.

(f) $\iint_R y \, dx \, dy$ where R is the interior part of the ellipse $\frac{x^2}{4} + \frac{y^2}{2} = 1$ with $y \geq 0$.

5. Use polar coordinates, taking $x = r\cos\theta$ and $y = r\sin\theta$, to evaluate each of the following integrals:

★(a) $\iint_R dx \, dy$ where R is the interior of the circle $x^2 + y^2 = 4$ Ans: 4π

(b) $\iint_R dx \, dy$ where R is the interior of the circle $(x-1)^2 + y^2 = 1$

(c) $\iint_R y(x^2 + y^2) \, dx \, dy$ where R is the interior of the semicircle $(x-1)^2 + y^2 = 1$ with $y \geq 0$