Introduction to Financial Mathematics - 20912

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Exercise Sheet 8 - Bond Pricing

1. (a) Show that the equation for the bond price

$$\frac{dV}{dt} = r(t)V - K(t)$$

has the explicit solution

$$V(t) = \exp\left(-\int_t^T r(s)ds\right)\left[F + \int_t^T K(y)\exp\left(\int_y^T r(s)ds\right)dy\right],$$

where $r(t)$ is the risk-free interest rate, $K(t)$ is the dividend payment, $V(T) = F$.

(b) When the interest rate r is constant, show that

$$V(t) = Fe^{-r(T-t)} + \int_t^T K(y)e^{-r(y-t)}dy.$$

2. (a) Find the bond price $V(t)$ at $t = 0$, when the payment $K(t) = 0$, and the interest rate $r(t)$ is

$$r(t) = \frac{r_1}{1+t} + \frac{tr_2}{1+t},$$

where r_1 and r_2 are constants.

Ans: $V(0) = F\left(1+T\right)^{(r_2-r_1)}\exp\left(-r_2T\right).$

(b) Find **the term structure of interest rate** $Y\left(0,T\right)$ for $r(t) = \frac{r_1}{1+t} + \frac{tr_2}{1+t}$ and plot $r(t)$ and $Y\left(0,T\right)$ when

1) $r_1 < r_2$;

2) $r_1 > r_2$.

Ans: $Y(0,T) = r_2 + (r_1 - r_2)\frac{1}{T}\ln\left(1+T\right).$

3. Find the bond price at $t = 0$, when the payment $K(t) = K = const$ and the interest rate $r(t) = r = const$.

Ans: $V(0) = e^{-rT}\left[F + K\frac{e^{rT}-1}{r}\right].$