# Introduction to Financial Mathematics - 20912

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**Exercise Sheet 7**

**1.** Show that the modified Black-Scholes equation

$$\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + (r - D_0)S\frac{\partial C}{\partial S} - rC = 0$$

has the explicit solution for the European call

$$C(S,t) = Se^{-D_0(T-t)}N\left(d_{10}\right) - Ee^{-r(T-t)}N\left(d_{20}\right),$$

where

$$d_{10} = \frac{\ln\left(S/E\right) + \left(r - D_0 + \sigma^2/2\right)(T-t)}{\sigma\sqrt{T-t}}, \; d_{20} = d_{10} - \sigma\sqrt{T-t}.$$

(Hint: use the explicit solution to the **Black-Scholes equation** and notes on options on dividend-paying assets)

**2.** Calculate the price of a three-month European call option on a stock with a continuous dividend yield $D_0 = 0.01$ and a strike price \$60 when the current stock price is \$80. The risk-free interest rate is 10% p.a. The volatility is 30%.

Ans: $C_0 = 21.349$

**3.** What is the delta for the call option with continuous dividend yield?

**4.** Show that the value of a European call option on a stock that pays a constant continuous dividend yield lies below the payoff for large enough values of $S$.