# Introduction to Financial Mathematics - 20912 <br> Lecturer - Prof. Sergei Fedotov 

## Exercise Sheet 5-American Put Option, Replicating Portfolio and Arbitrage.

1. A stock price is currently $\$ 40$. Over each of the next two 3-month periods it is expected to go up by $10 \%$ or down by $10 \%$. The risk-free interest rate is $12 \%$ per annum.
(a) What is the value of a 6 -month European put option with a strike price of $\$ 42$ ?
(Ans: 2.1183)
(b) What is the value of a 6 -month American put option with a strike price of $\$ 42$ ?
(Ans: 2.5372)
2. By using an one-step binomial model and the replicating portfolio $\Pi=\Delta S-N B$, where $\Delta$ is the number of shares, and $N$ is the number of bonds $B$, show that the value of an European call option is

$$
C_{0}=e^{-r T}\left[p C_{u}+(1-p) C_{d}\right]
$$

where

$$
p=\frac{e^{r T}-d}{u-d}
$$

$r$ is the risk-free interest rate, $T$ is the maturity time, $C_{u}$ is the payoff from the option if the stock price moves up, $C_{d}$ is the payoff from the option if the stock price moves down.
3. European call and put options are currently $\$ 5.09$ and $\$ 7.78$. The strike price is $\$ 24$. Exercise date $T=0.5$. The current price of the underlying stock is $\$ 20.37$ and the risk-free interest rate is $7.48 \%$. Find an arbitrage opportunity and corresponding profit.

Hint: use a put-call parity.

