Introduction to Financial Mathematics - 20912

Lecturer - Prof. Sergei Fedotov

Exercise Sheet 5 - American Put Option, Replicating Portfolio and Arbitrage.

1. A stock price is currently \$40. Over each of the next two 3-month periods it is expected to go up by 10% or down by 10%. The risk-free interest rate is 12% per annum.

(a) What is the value of a 6-month European put option with a strike price of \$42?

(Ans: 2.1183)

(b) What is the value of a 6-month **American** put option with a strike price of \$42?

(Ans: 2.5372)

2. By using an one-step binomial model and the replicating portfolio $\Pi = \Delta S - NB$, where Δ is the number of shares, and N is the number of bonds B, show that the value of an European call option is

$$C_0 = e^{-rT} \left[pC_u + (1-p)C_d \right],$$

where

$$p = \frac{e^{rT} - d}{u - d},$$

r is the risk-free interest rate, T is the maturity time, C_u is the payoff from the option if the stock price moves up, C_d is the payoff from the option if the stock price moves down.

3. European call and put options are currently \$5.09 and \$7.78. The strike price is \$24. Exercise date T = 0.5. The current price of the underlying stock is \$20.37 and the risk-free interest rate is 7.48%. Find an arbitrage opportunity and corresponding profit.

Hint: use a put-call parity.