Introduction to Financial Mathematics - 20912

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Exercise Sheet 4 - Binomial Model

1. *One-step binomial model.* A stock price is currently \$100. It is known that at the end of three months it will be either \$110 or \$90. The risk-free interest rate is 5% per annum with continuous compounding.

What is the value of three-month European call option with a strike price of \$95? Verify that *no-arbitrage* arguments and *risk-neutral valuation* give the same answer.

Ans: \$8.3385

2. *One-step binomial model.* A stock price is currently \$80. It is known that at the end of four months it will be either \$75 or \$85. The risk-free interest rate is 5% per annum with continuous compounding.

What is the value of four-month European put option with a strike price of \$80?

Hint: $P_0 = e^{-rT}\left[pP_u + (1-p)P_d\right]$.

Ans: \$1.7975

3. *Two-step binomial tree.* Stock price starts at \$20 and in each of the next two three-month periods may go up by 10% or down by 10%. The risk-free interest rate is 12% per annum with continuous compounding.

What is the value of six-month European call option with a strike price of \$21? (Hint: repeatedly apply the principles established for one-period model).

Ans: 1.2822

4. *Risk-Neutral World.* Consider a two-step binomial tree with $u = 1.2$ and $d = 0.9$. Initial stock price $S_0 = 40$. The interest rate is 5% p.a, compounded continuously.

Work out the probability distribution of stock price in six months in a risk-neutral world.

Ans: The stock price takes the values 57.6, 43.2 and 32.4 with the probabilities $0.141, 0.469$ and 0.390.

5 (not easy!). *Binomial tree: matching volatility σ with u and d.*

By using three equations

$$qu + (1-q)d = e^{\mu\Delta t}, \qquad qu^2 + (1-q)d^2 - (qu + (1-q)d)^2 = \sigma^2\Delta t, \qquad d = u^{-1},$$

find that

$$u = e^{\sigma\sqrt{\Delta t}} \approx 1 + \sigma\sqrt{\Delta t}, \qquad \mathit{Cox, Ross, Rubinstein} \quad (1979)$$

Use notes .