Introduction to Financial Mathematics - 20912

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Exercise Sheet 3 - No Arbitrage Principle, Upper and Lower Bounds

1. Using no-arbitrage arguments: $\Pi_T \geq 0$ then $\Pi_t \geq 0$ for $0 \leq t \leq T$, prove the following simple bounds on European call options with the exercise price E:

(a)

$$C_t \leq S_t, \qquad 0 \leq t \leq T.$$

where $C_t = C(S_t, t)$. Hint: Consider the portfolio $\Pi = S - C$ at time $t = T$.

(b)

$$S_t - E\exp(-r(T-t)) \leq C_t, \qquad 0 \leq t \leq T.$$

Hint: consider the portfolio $\Pi = C - S + B$, where B is the risk-free bond with face value E at time T .

(c) If two otherwise identical calls have strike prices E_1 and E_2 with $E_1 < E_2$, then

$$0 \leq C_t(E_1) - C_t(E_2) \leq E_2 - E_1.$$

Hint: consider the portfolio $\Pi = C(E_1) - C(E_2)$, where $C(E_1)$ and $C(E_1)$ are the values of calls with strike prices E_1 and E_2 respectively.

2. (a) Find a lower bound for the European call option with the exercise price £15 when the stock price is £21, the time to maturity is six months, and the risk-free interest rate is 8% per annum.

(b) Consider the situation where the European call option is £5 which is less that the theoretical minimum £6.588. Show that there exists an arbitrage opportunity.

Hint: consider the portfolio $\Pi = C + B - S$, where B is the risk-free bond with current value £16.

3. Show that a lower bound for an European put P_0 is

$$E\exp(-rT) - S_0.$$

Hint: use Put-Call parity.

4. The upper bound for an European put P_0 is

$$E\exp(-rT).$$

Show that if this is not true, then one can make a riskless profit (arbitrage opportunity). Hint: consider the portfolio $\Pi = -P + B$.