Introduction to Financial Mathematics - 20912

Lecturer - Prof. Sergei Fedotov

Exercise Sheet 1

1. (Warming up). What is the interest rate if a deposit subject to continuous compounding is doubled after 12 years? Ans: 5.78%.

2. (Warming up). What is the difference between £140 deposited at 10% compounded monthly ($m = 12$) and compounded continuously after one year? Ans: about £0.06

In what follows the stock price $S(t)$ obeys the stochastic differential equation

$$dS = \mu S dt + \sigma S dW,$$

where μ is the expected return, σ is the volatility, $W(t)$ is a standard Wiener process (Brownian motion).

3. Suppose that the expected return from a stock is 14% per annum and the volatility is 20%. Initial stock price is \$90. By using $dW \approx X\sqrt{\Delta t}$, where X is $N(0,1)$, calculate the increase ΔS in the stock price during three days.

Ans: $\Delta S \approx 0.104 + 1.632X$

4. Suppose that a stock price has an expected return of 36% per annum and a volatility of 40%. When the stock price at the end of a certain day is \$80, calculate the following

(a) the expected stock price at the end of the next day; Ans: 80.079;

(b) the standard deviation of the stock price at the end of the next day; Ans: 1.675

5. If $dS = \mu S dt + \sigma S dW$, and A and n are constants, find the SDE satisfied by (use Ito's Lemma)

(a) $f(S) = AS + t^2$, (Ans: $df = \left(2t + \mu\left(f - t^2\right)\right) dt + \sigma\left(f - t^2\right) dW$)

(b) $f(S) = S^{1/2}$, (Ans: $df = \left(\mu/2 - \sigma^2/8\right) f dt + \sigma/2 f dW$)

(c) $f(S) = S^n$ (Ans: $df = \left(\mu n + \frac{1}{2}\sigma^2 n(n-1)\right) f dt + \sigma n f dW$)

6. Suppose that initial stock price is \$90, the expected return from a stock is 24% per annum, and the volatility is 35%. Find the distribution of $\ln(S)$ in eight months' time.

Ans: $\ln S \sim N(4.619, 0.082)$

7. (**not easy!!**) Show that the probability density function $p(y,t)$ for the Wiener process $W(t)$

$$p(y,t) = \frac{\partial}{\partial y}\mathbb{P}\left(W(t) \leq y\right)) = \frac{1}{\sqrt{2\pi t}} \exp\left(-\frac{y^2}{2t}\right)$$

obeys the following differential equation

$$\frac{\partial p}{\partial t} = \frac{1}{2}\frac{\partial^2 p}{\partial y^2}\ .$$