Introduction to Financial Mathematics - 20912

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Solutions - 7

1. We have
$$C\left(S,t\right)=e^{-D_0(T-t)}C_1\left(S,t\right),$$
where $C_1\left(S,t\right)$ satisfies the classical Black-Scholes equation with r replaced by $r-D_0$. The value of $C_1\left(S,t\right)$ is therefore just that of a normal European call with interest rate $r-D_0$:
$$C_1(S,t)=SN\left(d_{10}\right)-Ee^{-(r-D_0)(T-t)}N\left(d_{20}\right),,$$
where
$$d_{10}=\frac{\ln\left(S/E\right)+\left(r-D_0+\sigma^2/2\right)\left(T-t\right)}{\sigma\sqrt{T-t}},\ d_{20}=d_{10}-\sigma\sqrt{T}.$$

2. In this case $D_0=0.01$, $S_0=80$, $E=60$, $T=0.25$, $\sigma=0.3$ and $r=0.1$. First, we compute the values of d_{10} and d_{20} at $t=0$:
$$d_{10}=\frac{\ln\left(S/E\right)+\left(r-D_0+\sigma^2/2\right)T}{\sigma\sqrt{T}}=\frac{\ln\left(\frac{80}{60}\right)+\left(0.1-0.01+\left(0.3\right)^2\times0.5\right)\right)\times0.25}{0.3\times\sqrt{0.25}}\approx2.1429$$
$$d_{20}=d_{10}-\sigma\sqrt{T-t}=2.1429-0.3\times\sqrt{0.25}\approx1.9929$$
The value of call option is
$$C_0=Se^{-D_0T}N\left(d_{10}\right)-Ee^{-rT}N\left(d_{20}\right).$$
Since
$$N\left(2.142\,9\right)\approx0.9839,\qquad N\left(1.992\,9\right)\approx0.9769$$
then
$$C_0=80\times e^{-0.01\times0.25}\times0.9839-60\times e^{-0.1\times0.25}\times0.9769\approx21.349$$

3. This exercise is very similar to one from problem sheet 6, part 4a.
$$\Delta=\frac{\partial C}{\partial S}=e^{-D_0(T-t)}N\left(d_{10}\right).$$

4. If $S\to\infty$, then $d_{10}\to\infty$ and $d_{20}\to\infty$. Therefore $N\left(d_{10}\right)\to1$ and $N\left(d_{20}\right)\to1$.

As $S\to\infty$, it follows from
$$C(S,t)=Se^{-D_0(T-t)}N\left(d_{10}\right)-Ee^{-r(T-t)}N\left(d_{20}\right)$$
that
$$C(S,t)\to Se^{-D_0(T-t)}.$$
For large S it is certainly below the payoff $C(S,T)=S-E\to S$, since $e^{-D_0(T-t)}$ is less than one.