Introduction to Financial Mathematics - 20912

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Exercise Sheet 2: Solutions

1 Equation for S(T) is $30 - S(T) - 2e^{0.1 \times \frac{1}{4}} = 4$. Solution: S(T) = 23.95

2 The expected gain or loss for a holder of a European call option is $\mathbb{E} \left[\max \left(S(T) - 94, 0 \right) \right] - 10e^{0.1 \times \frac{1}{2}} = \frac{1}{4} \times 0 + \frac{1}{4} \times (96 - 94) + \frac{1}{2} \times (98 - 94) - 10e^{0.1 \times \frac{1}{2}} = -8.01$

3. (a) Straddle:
$$\Pi = 2C - S = \begin{cases} -S, \ S \le E, \\ 2(S - E) - S, \ S > E, \end{cases} = \begin{cases} -S, \ S \le E, \\ S - 2E, \ S > E, \end{cases};$$

(b) Strip:
$$\Pi = 2P + C = \begin{cases} 2(E-S), \ S \le E, \\ S-E, \ S > E, \end{cases}$$
;

(c) Strap:
$$\Pi = P + 2C = \begin{cases} E - S, \ S \le E, \\ 2(S - E), \ S > E, \end{cases}$$
;

(d) Strangle: $\Pi = C(E_1) + P(E_2)$. Consider the case when $E_1 < E_2$

$$\Pi_T = \begin{cases} E_2 - S, \ S \le E_1, \\ E_2 - E_1, \ E_1 \le S < E_2, \\ S - E_1, \ E_2 \le S \end{cases} .$$

The case $E_1 > E_2$ is similar.

4. In a strip, the investor benefits when there is a large price move. The holder is betting that a decrease in the stock price to be more likely than an increase. The holder of a strap believes that increase in the stock price is more likely than a decrease.

5. One can work from left to write, starting by finding the number of call options with strike price 10, and then the number of call options with strike prices 20 and 30. The portfolio is

$$\Pi = 2C_{10} - 4C_{20} + 2C_{30}$$

. The butterfly spread is used to get a profit if the stock price stays close to 20. Investors is betting that large stock price moves are unlikely.

6. It is a similar to 5. The portfolio is

$$\Pi = C_{10} - C_{20} - C_{40} + C_{50}$$





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