# Introduction to Financial Mathematics - 20912 <br> Lecturer - Prof. Sergei Fedotov 

## Exercise Sheet 2: Solutions

1 Equation for $S(T)$ is $30-S(T)-2 e^{0.1 \times \frac{1}{4}}=4$. Solution: $S(T)=23.95$

2 The expected gain or loss for a holder of a European call option is $\mathbb{E}[\max (S(T)-94,0)]-10 e^{0.1 \times \frac{1}{2}}=$ $\frac{1}{4} \times 0+\frac{1}{4} \times(96-94)+\frac{1}{2} \times(98-94)-10 e^{0.1 \times \frac{1}{2}}=-8.01$
3. (a) Straddle: $\Pi=2 C-S=\left\{\begin{array}{c}-S, S \leq E, \\ 2(S-E)-S, S>E,\end{array} \quad=\left\{\begin{array}{c}-S, S \leq E, \\ S-2 E, S>E,\end{array}\right.\right.$;
(b) Strip: $\quad \Pi=2 P+C=\left\{\begin{array}{c}2(E-S), S \leq E, \\ S-E, S>E,\end{array}\right.$;
(c) $\quad$ Strap: $\quad \Pi=P+2 C=\left\{\begin{array}{c}E-S, S \leq E, \\ 2(S-E), S>E,\end{array}\right.$;
(d) Strangle: $\Pi=C\left(E_{1}\right)+P\left(E_{2}\right)$. Consider the case when $E_{1}<E_{2}$

$$
\Pi_{T}=\left\{\begin{array}{c}
E_{2}-S, S \leq E_{1} \\
E_{2}-E_{1}, E_{1} \leq S<E_{2} \\
S-E_{1}, E_{2} \leq S
\end{array}\right.
$$

The case $E_{1}>E_{2}$ is similar.
4. In a strip, the investor benefits when there is a large price move. The holder is betting that a decrease in the stock price to be more likely than an increase. The holder of a strap believes that increase in the stock price is more likely than a decrease.
5. One can work from left to write, starting by finding the number of call options with strike price 10 , and then the number of call options with strike prices 20 and 30 . The portfolio is

$$
\Pi=2 C_{10}-4 C_{20}+2 C_{30}
$$

. The butterfly spread is used to get a profit if the stock price stays close to 20 . Investors is betting that large stock price moves are unlikely.
6. It is a similar to $\mathbf{5}$. The portfolio is

$$
\Pi=C_{10}-C_{20}-C_{40}+C_{50}
$$

(4)

(c)

(d)

$$
E_{1}<E_{2}
$$

$$
E_{1}>E_{2}
$$




