Lecture 5

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20912 - Introduction to Financial Mathematics

Lecture 5

1. Trading Strategies: Straddle, Bull Spread, etc.

2. Bond and Risk-Free Interest Rate

3. No Arbitrage Principle

Portfolio and Short Selling

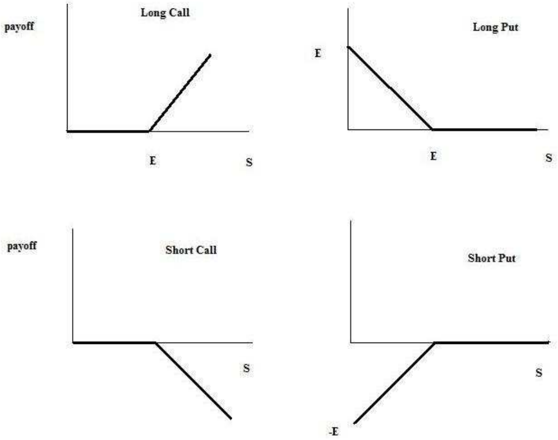
Reminder from previous lecture 4.

Definition. Short selling is the practice of selling assets that have been borrowed from a broker with the intention of buying the same assets back at a later date to return to the broker. This technique is used by investors who try to profit from the falling price of a stock.

Definition. Portfolio is the combination of assets, options and bonds. We denote by Π the value of portfolio.

Examples.

Option positions



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Bull Spread

Bull spread is a strategy that is designed to profit from a moderate rise in the price of the underlying security.

Let us set up a portfolio consisting of a long position in call with strike price $E_1$and short position in call with E_2 such that $E_1 < E_2$.

The value of this portfolio is $\Pi_t = C_t\left(E_1\right) - C_t\left(E_2\right).$ At maturity $t = T$

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• The holder of this portfolio benefits when the stock price will be above E_1.

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If $B(T) = F$, then $B(t) = Fe^{-r(T-t)}$, where $e^{-r(T-t)}$ - discount factor

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Arbitrage opportunities may exist in a real market. But, they cannot last for a long time.