Lecture 14

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20912 - Introduction to Financial Mathematics

1. Δ-Hedging

2. Greek Letters or Greeks

Delta for European Call Option

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$$N(d_1) + (SN'(d_1) - Ee^{-r(T-t)}N'(d_2))\frac{\partial d_1}{\partial S}.$$

We need to prove

$$(SN'(d_1) - Ee^{-r(T-t)}N'(d_2)) = 0.$$

See Problem Sheet 6.

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We find

$$d_1 = \frac{(0.06 + (0.16)^2 \times 0.5)) \times 0.5}{0.16 \times \sqrt{0.5}} \approx 0.3217$$

and

$$\Delta = N(0.3217) \approx 0.6262$$

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We find $1 + \frac{\partial P}{\partial S} - \frac{\partial C}{\partial S} = 0.$

Therefore $\frac{\partial P}{\partial S} = \frac{\partial C}{\partial S} - 1 = N(d_1) - 1.$

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Problem Sheet 6: $\Gamma = \frac{\partial^2 C}{\partial S^2} = \frac{N'(d_1)}{S\sigma\sqrt{T-t}}$, where $N'(d_1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}}$.

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The Greeks are important tools in financial risk management. Each Greek measures the sensitivity of the value of derivatives or a portfolio to a small change in a given underlying parameter.