MATH10131

TWO AND A HALF HOURS

THE UNIVERSITY OF MANCHESTER

CALCULUS AND VECTORS

DATE: 20 January 2017

TIME: 14.00 - 16.30

Answer ALL FIFTEEN questions

Electronic calculators and formula tables are not permitted.

No prepared notes of any kind are to be brought into the examination room.

This examination makes up 75% of the overall assessment for this course unit.

P.T.O.

1. Find the set of real numbers x satisfying $|x^2 - 4| < 2$. [2]

2. Sketch graphs of the following real-valued functions of x satisfying

(a) $f(x) = 2^{-|x+1|} + 1$; (b) $f(x) = |\ln(2 - x) - \ln 2|$. [4]

3. By using implicit differentiation, find the tangent line to the curve defined by the equation

$$x^3 + y^3 = 6xy$$

at the point $(3, 3)$. [4]

4. Find the Maclaurian series for the function

$$\sinh(x).$$

[4]

5. Find the derivative of the function

$$g(x) = \sqrt{1 + 2x}$$

by using the definition of derivative

$$g'(x) = \lim_{h \to 0} \frac{g(x + h) - g(x)}{h}.$$

[4]

6. Find the limits

(a) $\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{\sin(x)} \right)$; (b) $\lim_{x \to 0^+} x^{x^2}$. [6]

7. Find the Taylor series for the function

$$g(x) = (1 + 2x)^2$$

at $a = 2$. [4]

8. Find the area of the region bounded by the curves $y = \sin(2x)$, $y = \cos(x)$ and the lines $x = 0$, $x = \pi/2$. [6]

9. Find the area enclosed by the ellipse

$$x^2 + \frac{y^2}{4} = 1.$$

[6]

10. Evaluate the definite integrals

(a) $\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$, (b) $\int_0^a x\sqrt{x^2 + a^2}dx$, $a > 0$. [4]

11. Find an equation of the plane that contains the line $x = 3+2t$, $y = t$, $z = 8 - t$ and is parallel to the plane $2x + 4y + 8z = 17$. [4]

12. Where does the line through the points $(1, 0, 1)$, $(4, -2, 2)$ intersect the plane $x + y + z = 6$. [6]

13. Find the area of the region that lies inside the curve $r = 1 + \cos\theta$, $0 \leq \theta \leq 2\pi$ and outside the curve $r = 3\cos\theta$, $-\pi/2 \leq \theta \leq \pi/2$. [6]

14. Evaluate the double integral

$$\int\int_D \sqrt{4 - x^2 + y^2}dxdy,$$

where $D = \{(x, y) \mid x^2 + y^2 \leq 4, x > 0\}$. [6]

15. Evaluate the line integral $\int_C \vec{f} \cdot d\vec{r}$ of the vector field

$$\vec{f} = (2x, -2y),$$

where the plane curve C is
(a) one quarter of the circle $x^2 + y^2 = 1$ from $(0, 1)$ to $(1, 0)$,
(b) a straight line from $(0, 1)$ to $(1, 0)$. [9]

END OF EXAMINATION PAPER