Lecture 8

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10131 - Calculus and Vectors

Implicit differentiation. Maximum and minimum values.

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1. Derivatives of inverse trigonometric and logarithmic functions

2. Logarithmic differentiation

3. Maximum and minimum values, second derivative test

Derivatives of inverse trigonometric and logarithmic functions

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$$\frac{d}{dx}\log_a x = \frac{1}{x \ln a}.$$

Solution: We find the derivatives of $\log_a x$ by using implicit differentiation. Let $y = \log_a x$. It means that $a^y = x$. We differentiate this equation implicitly with respect to x and obtain...

Logarithmic differentiation.

Example: Find the derivative

$$y = \frac{\sqrt{x^2+1}}{(3x+1)^5}.$$

Solution: We take logarithms of both sides of this equation:

$$\ln y = \frac{1}{2}\ln(x^2+1) - 5\ln(3x+1)$$

Differentiating implicitly wrt x, we obtain...

Maximum and minimum values

Definition. A function f has an absolute maximum at c if $f(x) \leq f(c)$ for all x in the domain D of f. The number $f(c)$ is called the maximum value of f in D. Similarly, f has an absolute minimum at c if $f(c) \leq f(x)$.

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Definition. A stationary (critical) point of f is a point c for which

$$f'(c) = 0$$

Second derivative test

Suppose that the second derivative $f''(x)$ is continuous near c.

1) If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c;

2) If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c.

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Example: Find the stationary points and determine their nature for the function:

$$f(x) = \frac{x^3}{3} - \frac{x^2}{2} - 2x + 3.$$

Example: Sketch the graph of

$$y = \frac{x}{x^2 + 1}.$$