Lecture 7

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10131 - Calculus and Vectors

Differentiation Rules

Lecture 7

1. Chain rule
2. L'Hospital rule
3. Implicit differentiation

Chain Rule

The Chain Rule. If $y = f(u)$ and $u = g(x)$ are differentiable functions, then

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

or

$$\frac{d}{dx}f\left(g(x)\right) = f'\left(g(x)\right) \times g'(x).$$

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Example: Differentiate the following functions

$$y = \sqrt{1 + x^3}, \quad y = \sin^2 x, \quad y = e^{\sin^5 x}.$$

We observe that these are the composite functions. If we let $u = g(x) = 1 + x^3$, then we can write $y = f(u) = \sqrt{u}$

L'Hospital rule

L'Hospital's Rule. We assume that the functions $f(x)$ and $g(x)$ are differentiable, the derivative $g'(x) \neq 0$ and

$$\lim_{x \to a} f(x) = 0 \qquad \lim_{x \to a} g(x) = 0$$

or

$$\lim_{x \to a} f(x) = \pm\infty \qquad \lim_{x \to a} g(x) = \pm\infty$$

Then

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Examples: Find the following limits:

$$\lim_{x\to 2} \frac{\ln(x-1)}{x-2}, \qquad \lim_{x\to\infty} \frac{e^x}{x^2}$$

Higher derivatives.

$f'(x)$ is also a function, so $f'(x)$ may have a derivative of its own.
Notation for the n^{th} derivative:

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Implicit differentiation.

Example: For $x^2 + y^2 = 9$, find $\frac{dy}{dx}$ and an equation of the tangent line to the circle at point $(2, \sqrt{5})$.