Lecture 6

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10131 - Calculus and Vectors

Derivatives

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1. Interpretation of the derivative as a slope of tangent line
2. Interpretation of the derivative as a rate of change

Tangent line to curve $y = f(x)$

Motivation: The problem of finding the tangent line to a curve. Let us assume that the curve C is described by the equation $y = f(x)$. Our aim is to find the tangent line to C at the point $(a, f(a))$.

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The first step is to compute the slope of the secant line

$$m = \frac{f(x) - f(a)}{x - a}.$$

Equation for secant line $y = m(x - a) + f(a)$.

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We write an equation of the tangent line to the curve $y = f(x)$ at the point $(a, f(a))$ as follows

$$y = f'(a)(x - a) + f(a), \qquad f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

where $f'(a)$ is a derivative of a function f at a if this limit exists.

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The tangent line to $y = f(x)$ at the point $(a, f(a))$ is the line through $(a, f(a))$ whose slope is equal to the derivative $f'(a)$.

Alternative expression for the derivative

If $h = x - a$, then $x = a + h$ and the derivative at point a can be written as follows

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Definition. The derivative of a function $f(x)$ at point x is

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Example: Find the derivative of the function $f(x) = \frac{2}{x}$ from the definition of $f'(x)$.

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Example: If $f(x) = x^2 + 1$, find a formula for $f'(x)$ and illustrate by comparing the graphs of $y = f(x)$ and $y = f'(x)$.

Example: Find an equation of the tangent line to the parabola $y = -x^2 + 5$ at the point $(2, 1)$.

Interpretation of the derivative as a rate of change

Quotient:

$$\frac{\Delta y}{\Delta x} = \frac{f(x+h) - f(x)}{h}$$

is called the average rate of change of $f(x)$ with respect to x.

Derivative:

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Example: The position of a particle is given by the equation $s(t) = 2 + t^3$. Find the velocity of a particle at $t = 2$.

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If we use the notation $y = f(x)$, then some common alternative notations for the derivative are as follows

$$f'(x), \quad y', \quad \frac{dy}{dx}, \quad \frac{df}{dx}, \quad \frac{d}{dx}f(x).$$