Lecture 30

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10131 - Calculus and Vectors

Line Integrals

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1. Parametric equations of curves
2. Line integral

Line integral

Let us start with a plane curve C given by the parametric equations

$$\overrightarrow{\mathbf{r}} = \overrightarrow{\mathbf{r}}(t).$$

or

$$x = x(t), \quad y = y(t), \quad a \leq t \leq b$$

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We define the line integral of $\overrightarrow{\mathbf{f}}$ along a smooth curve C as

$$\int_C \overrightarrow{\mathbf{f}} \cdot d\overrightarrow{\mathbf{r}} = \int_a^b \overrightarrow{\mathbf{f}}\left(\overrightarrow{\mathbf{r}}(t)\right) \cdot d\overrightarrow{\mathbf{r}}'(t)dt.$$

Examples

Example 1: Find the line integral of

$$\overrightarrow{\mathbf{f}} = \nabla \left(x^2 - y^2 \right)$$

on one quarter of the circle

$$x^2 + y^2 = 1$$

from $(0, 1)$ to $(1, 0)$.

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Example 2: Find the line integral of

$$\overrightarrow{\mathbf{f}} = \nabla \left(x^2 - y^2 \right)$$

on a straight line from $(0, 1)$ to $(1, 0)$.