Lecture 3

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10131 - Calculus and Vectors

Exponential and logarithmic functions

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1. Inverse functions
2. Exponential and logarithmic functions
3. Sketching the graphs
4. Hyperbolic functions

Inverse functions

The graph of f^{-1} is obtained by reflecting the graph of f about the line $y = x$.

Example: Sketch the graph of $f(x) = \sqrt{-x}$ and its inverse.

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Exponential functions

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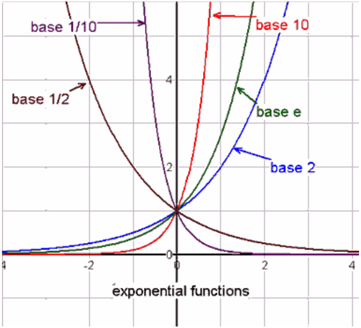

Logarithmic functions

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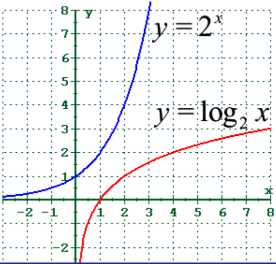

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Hyperbolic functions

Hyperbolic functions are defined by using the exponential functions

$$\sinh x = \frac{e^x - e^{-x}}{2}, \quad \cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{\sinh x}{\cosh x}, \quad \coth x = \frac{\cosh x}{\sinh x}$$

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Example: Sketch the graph of the function $\cosh x$ and determine the domain and range