# Lecture 29

Lecturer: Prof. Sergei Fedotov

10131 - Calculus and Vectors

## Change of Variables in Double Integrals

# Lecture 29

1. Jacobian
2. Change of Variables in Double Integrals

Definition. The Jacobian of the transformation $T$ given by

$$x = x(u, v), \qquad y = y(u, v)$$

is

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u}$$

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We assume that $T$ is the one-to-one transformation with nonzero Jacobian and that maps a region $S$ in the $uv$-plane onto a region $R$ in the $xy-$plane. Suppose that $f$ is continuous on $R$ then

$$\int\int_R f(x, y)dA = \int\int_S f(x(u, v), y(u, v)) \left| \frac{\partial(x, y)}{\partial(u, v)} \right| dudv.$$

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Note that $dA = \left|\frac{\partial(x,y)}{\partial(u,v)}\right| dudv$ and $\frac{\partial(x,y)}{\partial(u,v)} = 1/\frac{\partial(u,v)}{\partial(x,y)}$.

# Examples

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Example 3. Find $\int\int_D f(x,y)dA$, where $D$ is the domain between the four curves

$$y = \frac{1}{x}, \quad y = \frac{3}{x}, \quad y = \frac{x}{e}, \quad y = ex.$$

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We define

$$a = xy \quad b = \frac{y}{x}$$