Lecture 28

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10131 - Calculus and Vectors

Double Integrals in Polar Coordinates

1. Change to Polar Coordinates in a Double Integrals (Polar Rectangle)
2. General Domain

Change to Polar Coordinates in a Double Integrals (Polar Rectangle)

If we consider the polar rectangle

$$R = \{(r, \theta) | \ a \leq r \leq b, \ \alpha \leq \theta \leq \beta\}$$

then for the continuous function f on R

$$\int\int_R f(x, y) dA = \int_\alpha^\beta \int_a^b f(r\cos\theta, r\sin\theta) r dr d\theta.$$

Change to Polar Coordinates in a Double Integrals (Polar Rectangle)

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Example:

Find the volume of the solid bounded by the plane $z = 0$ and the paraboloid

$$z = 1 - x^2 - y^2.$$

General Domain

If we consider the general domain

$$D = \{(r, \theta) | \alpha \leq \theta \leq \beta, h_1(\theta) \leq r \leq h_2(\theta)\}$$

then for the continuous function f on D

$$\int\int_D f(x, y) dA = \int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} f(r\cos\theta, r\sin\theta) r dr d\theta.$$

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Example.

Use a double integral to find the area enclosed by one loop of the four-leaved rose

$$r = \cos 2\theta$$

for $-\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$.