Lecture 27

Lecturer: Prof. Sergei Fedotov

10131 - Calculus and Vectors

Double Integrals (examples)

Double Integrals over Domain of Type I

The plane region D of type I:
$D = \left\{(x, y) \in \mathbb{R}^2 |\ a \leq x \leq b,\ g_1(x) \leq y \leq g_2(x)\right\}.$

For continuous function f on region D, we have

$$\int\int_D f(x, y)dA = \int_a^b \left[\int_{g_1(x)}^{g_2(x)} f(x, y)dy\right] dx.$$

Example:

Evaluate the double integral

$$\int\int_D xydxdy$$

over the triangle D with vertices

$$(0, 0), \quad (1, 0), \quad (1, 1).$$

Double Integrals over Domain of type II

The plane region D of type II:
$D = \left\{(x, y) \in \mathbb{R}^2 |\ c \leq y \leq d,\ h_1(y) \leq x \leq h_2(y)\right\}$.

For continuous function f on D, we have

$$\int\int_D f(x, y))dA = \int_c^d \left[\int_{h_1(y)}^{h_2(y)} f(x, y)dx\right] dy$$

Example.

Evaluate the double integral

$$\int\int_D (2x - y^2)dxdy$$

over the triangular region D enclosed between the lines

$$y = -x + 1, \quad y = x + 1, \quad y = 3.$$

Example

Find the volume of the solid that lies under the paraboloid

$$z = x^2 + y^2$$

and above the region D in the xy-plane bounded by the line

$$y = 2x$$

and the parabola

$$y = x^2.$$