Lecture 26

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10131 - Calculus and Vectors

Double Integrals

Lecture 26

1. Double integral of $f(x, y)$ over the rectangle
2. Double Integrals over General Domains

Double integral of $f(x, y)$ over the rectangle

We define a closed rectangle

$$R = \left\{(x, y) \in \mathbb{R}^2 |\ a \leq x \leq b,\ c \leq y \leq d\right\}.$$

Suppose that $f(x, y) > 0$. Let S be a solid that lies under the surface $z = f(x, y)$ and above R, then the double integral of $f(x, y)$ over the rectangle R

$$\int\int_R f(x, y)dA \quad or \quad \int\int_R f(x, y)dxdy$$

is the volume of this solid.

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Now we express a double integral as an iterated integral

$$\int\int_R f(x, y)dA = \int_a^b \left[\int_c^d f(x, y)dy\right] dx.$$

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or

$$\int\int_R f(x, y)dA = \int_c^d \left[\int_a^b f(x, y)dx\right] dy.$$

Example

Evaluate the double integral

$$\int\int_R xydA$$

over the rectangle

$$R = \left\{(x, y) \in \mathbb{R}^2 |\ 0 \leq x \leq 1,\ 0 \leq y \leq 2\right\}.$$

Double Integrals over General Domains

Double Integral over D:

$$\int\int_D f(x,y)dA$$

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Double Integral over D:

$$\int\int_D f(x,y)dA$$

The plane region D of type I:

$$D = \left\{(x,y) \in \mathbb{R}^2 |\ a \leq x \leq b,\ g_1(x) \leq y \leq g_2(x)\right\}.$$

For continuous function f on a type I region D, we have

$$\int\int_D f(x,y)dA = \int_a^b \left[\int_{g_1(x)}^{g_2(x)} f(x,y)dy\right] dx.$$

Double Integrals over General Domains of type II

The plane region D of type II:

$$D = \left\{(x, y) \in \mathbb{R}^2 |\ c \leq y \leq d,\ h_1(y) \leq x \leq h_2(y)\right\}.$$

For continuous function f on a type II region D, we have

$$\int\int_D f(x, y)dA = \int_c^d \left[\int_{h_1(y)}^{h_2(y)} f(x, y)dx\right] dy$$

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For continuous function f on a type II region D, we have

$$\int\int_D f(x, y)dA = \int_c^d \left[\int_{h_1(y)}^{h_2(y)} f(x, y)dx\right] dy$$

Example.

The region D in the xy-plane bounded by the line $y = 2x$ and the parabola $y = x^2$.