# Lecture 23

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10131 - Calculus and Vectors

**Tangent plane and chain rules**

# Lecture 23

1. Linear approximation of $f(x, y)$
2. Differential
3. Chain rules

# Linear approximation and tangent plane

Linear approximation of a function $f$ of two variables at a point $(a, b)$ :

$$f(x, y) \simeq f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b).$$

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An equation for the tangent plane to the surface $z = f(x, y)$ at the point $(a, b, f(a, b))$ is

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Example: Find an equation of the tangent plane to the paraboloid

$$z = \frac{x^2 + y^2}{2}$$

at the point $(1, 1, 1)$ .

# Differential and chain rule

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Chain rule (case 1). If $z = f(x, y)$ is a differentiable function of $x$ and $y$, where $x = g(t)$ and $y = h(t)$ are differentiable functions of $t$, then

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Example: Find $\frac{dz}{dt}$ for $z = x^2 + y^2$, where $x = \sin(2t)$ and $y = \cos(2t)$.