Lecture 21

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10131 - Calculus and Vectors

Distance of the point from the lines and planes

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1. Symmetric equations for line
2. Distance from a point to the line and planes

Symmetric equations for line

Recall that symmetric equations for line are

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}.$$

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Examples.

1. Find the symmetric equations for the line of intersection L between two planes

$$x + y + z = 1, \qquad x + y - z = 1.$$

Solution: Since L lies in both planes, it is perpendicular to both of normal vectors. Thus the vector $\overrightarrow{\mathbf{v}} = (a, b, c)$ can be found as a cross product of these normal vectors.

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2. Find the angle between two planes

$$x+y+z=1, \qquad x+y-z=1.$$

Show that the distance l of the point $\overrightarrow{\mathbf{P}}$ from the line $\overrightarrow{\mathbf{r}} = \overrightarrow{\mathbf{r}_0} + t\,\overrightarrow{\mathbf{v}}$ is

$$l = \frac{\left|(\overrightarrow{\mathbf{P}} - \overrightarrow{\mathbf{r}_0}) \times \overrightarrow{\mathbf{v}}\right|}{|\overrightarrow{\mathbf{v}}|}.$$

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Example: Find the distance from the point $\overrightarrow{\mathbf{P}} = (0, 1, 0)$ to the line $x(t) = 2 + t,\ y(t) = 1 - t,\ z(t) = t.$

Distance form the point to the line and plane

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Example: Find the distance from the point $\overrightarrow{\mathbf{P}} = (1, 1, 3)$ to the $x - z$ plane (the plane containing the x and z axes).