Lecture 20

Lecturer: Prof. Sergei Fedotov

10131 - Calculus and Vectors

Equations of Lines and Planes

Lecture 20

1. Vector equation of the line
2. Parametric and symmetric equations
3. Vector equation of the plane

Vector equation for the line

Vector equation for the line L through the point (x_0, y_0, z_0) and parallel to the vector $\overrightarrow{\mathbf{v}} = (a, b, c)$ is

$$\overrightarrow{\mathbf{r}} = \overrightarrow{\mathbf{r}_0} + t\,\overrightarrow{\mathbf{v}},$$

where $t \in \mathbb{R}$.

Vector equation for the line

Vector equation for the line L through the point (x_0, y_0, z_0) and parallel to the vector $\overrightarrow{\mathbf{v}} = (a, b, c)$ is

$$\overrightarrow{\mathbf{r}} = \overrightarrow{\mathbf{r}_0} + t\,\overrightarrow{\mathbf{v}},$$

where $t \in \mathbb{R}$.

Since $\overrightarrow{\mathbf{r}} = (x, y, z)$, one can write the parametric equations as follows

$$x = x_0 + at, \qquad y = y_0 + bt, \qquad z = z_0 + ct.$$

Each value of the parameter t gives the point (x, y, z)

Vector equation for the line

Vector equation for the line L through the point (x_0, y_0, z_0) and parallel to the vector $\overrightarrow{\mathbf{v}} = (a, b, c)$ is

$$\overrightarrow{\mathbf{r}} = \overrightarrow{\mathbf{r}_0} + t\,\overrightarrow{\mathbf{v}},$$

where $t \in \mathbb{R}$.

Since $\overrightarrow{\mathbf{r}} = (x, y, z)$, one can write the parametric equations as follows

$$x = x_0 + at, \qquad y = y_0 + bt, \qquad z = z_0 + ct.$$

Each value of the parameter t gives the point (x, y, z)

Example: Find a vector equation and parametric equations for the line through the point $(-2, 4, -3)$ and parallel to the vector $-\overrightarrow{\mathbf{i}} + \overrightarrow{\mathbf{j}}$.

Symmetric equations for line and vector equation of plane

If we eliminate the parameter t from the parametric equations, we obtain symmetric equations for line

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}.$$

Symmetric equations for line and vector equation of plane

If we eliminate the parameter t from the parametric equations, we obtain symmetric equations for line

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}.$$

A plane in space is determined by a point (x_0, y_0, z_0) on the plane and vector $\overrightarrow{\mathbf{n}}$ orthogonal to the plane. Vector equation of the plane:

$$\left(\overrightarrow{\mathbf{r}} - \overrightarrow{\mathbf{r}_0}\right) \cdot \overrightarrow{\mathbf{n}} = 0.$$

Scalar equation

Scalar equation of the plane through the point (x_0, y_0, z_0) with normal vector $\overrightarrow{\mathbf{n}} = (a, b, c)$ is

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0.$$

Scalar equation

Scalar equation of the plane through the point (x_0, y_0, z_0) with normal vector $\overrightarrow{\mathbf{n}} = (a, b, c)$ is

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0.$$

Examples:

Find the equation of the plane through the three points
$(1, 1, 1), \quad (0, 1, 2), \quad (-1, 1, -1).$