Lecture 2

Lecturer: Prof. Sergei Fedotov

10131 - Calculus and Vectors

Complex plane, functions

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1. Complex plane and set $\mid z \mid = 1$.
2. Functions (domain, range, rigorous notation, etc.)
3. Inverse functions

Complex plane

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Solution: $\mid z - 1 \mid < 2$ can be written as $\sqrt{(a-1)^2 + b^2} < 2$ or $(a-1)^2 + b^2 < 4$. That is a space inside of circle with radius 2 and centre at $z = 1$.

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Example: Sketch, in the complex plane, where $\mid z - 3 + i \mid < 1$.

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(c) $g(t) = \frac{5}{t^2 - t}$.

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Example: Find the inverse function of $f(x) = x^2 + 3$ for $x > 0$.