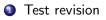
Lecture 19

Lecturer: Prof. Sergei Fedotov

10131 - Calculus and Vectors

Test revision + Cross Product



- Oross product as determinant
- Scalar triple products and volume of parallelepiped

Cross product

We can write the cross product as

$$\overrightarrow{\mathbf{a}} imes \overrightarrow{\mathbf{b}} = egin{bmatrix} \overrightarrow{\mathbf{i}} & \overrightarrow{\mathbf{j}} & \overrightarrow{\mathbf{k}} \ a_1 & a_2 & a_3 \ b_1 & b_2 & b_3 \end{bmatrix}$$

where

$$\begin{vmatrix} \overrightarrow{\mathbf{i}} & \overrightarrow{\mathbf{j}} & \overrightarrow{\mathbf{k}} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \overrightarrow{\mathbf{i}} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \overrightarrow{\mathbf{j}} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \overrightarrow{\mathbf{k}}$$

Example

Find the cross product of two vectors

$$\overrightarrow{\mathbf{a}} = (3, 2, -1)$$
 and $\overrightarrow{\mathbf{b}} = (1, -1, 1)$

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Scalar triple product of the three vectors $\overrightarrow{a}, \ \overrightarrow{b}$ and \overrightarrow{c} is

$$\vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

Scalar triple product of the three vectors \overrightarrow{a} , \overrightarrow{b} and \overrightarrow{c} is

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The volume V of the parallelepiped determined by three vectors $\vec{\mathbf{a}}$, $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ is the magnitude of their scalar triple product $V = \left| \vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) \right|$.

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Area of triangle ABC:

$$\frac{1}{2}\left|\overrightarrow{\mathbf{AB}}\times\overrightarrow{\mathbf{AC}}\right| = \frac{1}{2}|\overrightarrow{\mathbf{AB}}||\overrightarrow{\mathbf{AC}}|\sin\theta,$$

where $\theta = \angle CAB$

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Area of triangle ABC:

$$\frac{1}{2} \left| \overrightarrow{\mathbf{AB}} \times \overrightarrow{\mathbf{AC}} \right| = \frac{1}{2} |\overrightarrow{\mathbf{AB}}| |\overrightarrow{\mathbf{AC}}| \sin \theta,$$

where $\theta = \angle CAB$

Example. Find the area of the triangle with vertices at $\overrightarrow{\mathbf{A}} = (3, 3, 3)$, $\overrightarrow{\mathbf{B}} = (1, 1, 1)$ and $\overrightarrow{\mathbf{C}} = (5, 4, 3)$.

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