Lecture 17

Lecturer: Prof. Sergei Fedotov

10131 - Calculus and Vectors

Vectors

Lecture 17

1. Vectors (components, magnitude)
2. Properties of vectors
3. The dot and cross products

Vector and vector addition

A vector quantity has both length (magnitude) and direction. The opposite is a scalar quantity, which only has magnitude. Vectors can be denoted by $\overrightarrow{\mathbf{AB}}$ or $\overrightarrow{\mathbf{a}}$. Graphically, a vector is represented by an arrow, defining the direction, and the length of the arrow defines the vector's magnitude.

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We can define vector addition as follows. The sum of two vectors, $\overrightarrow{\mathbf{a}}$ and $\overrightarrow{\mathbf{b}}$, is a vector $\overrightarrow{\mathbf{c}}$, which is obtained by placing the initial point of $\overrightarrow{\mathbf{b}}$ on the final point of $\overrightarrow{\mathbf{a}}$, and then drawing a line from the initial point of $\overrightarrow{\mathbf{a}}$ to the final point of $\overrightarrow{\mathbf{b}}$ (Triangle Law)

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This addition method is sometimes called the Parallelogram Law because two vectors $\overrightarrow{\mathbf{a}}$ and $\overrightarrow{\mathbf{b}}$ form the sides of a parallelogram and two vectors $\overrightarrow{\mathbf{a}} + \overrightarrow{\mathbf{b}}$ is one of the diagonals.

Magnitude and components

It is convenient to introduce a coordinate system and treat vectors algebraically. We write

$$\overrightarrow{\mathbf{a}} = (a_1, a_2, a_3) \quad in \quad \mathbb{R}^3.$$

Here $\mathbb{R}^3 = \{(x, y, z)|_{x,y,z\in\mathbb{R}}\}$ is the set of all triples of real numbers.
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$$\overrightarrow{\mathbf{a}} + \overrightarrow{\mathbf{b}} = (a_1 + b_1, a_2 + b_2, a_3 + b_3), \quad \overrightarrow{\mathbf{a}} - \overrightarrow{\mathbf{b}} = (a_1 - b_1, a_2 - b_2, a_3 - b_3).$$

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Scalar multiplication:

$$c\overrightarrow{\mathbf{a}} = (ca_1, ca_2, ca_3)$$

Dot product

The dot product of two vectors $\overrightarrow{\mathbf{a}}$ and $\overrightarrow{\mathbf{b}}$ (the inner product, or the scalar product) is denoted by $\overrightarrow{\mathbf{a}} \cdot \overrightarrow{\mathbf{b}}$ and is defined as:

$$\overrightarrow{\mathbf{a}} \cdot \overrightarrow{\mathbf{b}} = |\overrightarrow{\mathbf{a}}||\overrightarrow{\mathbf{b}}| \cos\theta,$$

where θ is the angle between $\overrightarrow{\mathbf{a}}$ and $\overrightarrow{\mathbf{b}}$.

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Two vectors $\overrightarrow{\mathbf{a}}$ and $\overrightarrow{\mathbf{b}}$ are orthogonal if and only if $\overrightarrow{\mathbf{a}} \cdot \overrightarrow{\mathbf{b}} = 0$.

Cross product

The cross product (also called the vector product) differs from the dot product. The result of the cross product of two vectors $\overrightarrow{\mathbf{a}}$ and $\overrightarrow{\mathbf{b}}$ is a vector. The cross product, denoted by $\overrightarrow{\mathbf{a}} \times \overrightarrow{\mathbf{b}}$, is a vector perpendicular to both $\overrightarrow{\mathbf{a}}$ and $\overrightarrow{\mathbf{b}}$.
The magnitude of is

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The magnitude of $\overrightarrow{\mathbf{a}} \times \overrightarrow{\mathbf{b}}$ is the area of the parallelogram having $\overrightarrow{\mathbf{a}}$ and $\overrightarrow{\mathbf{b}}$ as sides.