Lecture 16

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10131 - Calculus and Vectors

Polar coordinates

Lecture 16

1. Polar coordinates
2. Graph of a polar equation
3. Areas in polar coordinates

Polar coordinates

The polar coordinates is a two-dimensional coordinate system in which each point on a plane is determined by a distance from a fixed point and an angle from a fixed direction.

The fixed point (analogous to the origin of a Cartesian system) is called the pole, and the ray from the pole in the fixed direction is the polar axis. The distance from the pole is called the radial coordinate or radius, and the angle is the angular coordinate, polar angle.

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Polar curves. The graph of a polar equation:

$$r = f(\theta), \quad \textit{or} \quad F(r,\theta) = 0.$$

Example: Sketch the curve

$$r = 2\cos\theta, \qquad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}.$$

Polar curves

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Polar rose is a famous mathematical curve that looks like a flower, and it can be expressed as a simple polar equation:

$$r(\theta) = \cos(k\theta)$$

If k is an integer, these equations will produce a k-petaled rose if k is odd, or a $2k$-petaled rose if k is even.

Areas in polar coordinates (Problem Sheet 8, 4)

An area element in polar coordinates

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The region bounded by $r = r(\theta)$ and the rays $\theta = \alpha$ and $\theta = \beta$ ($\alpha < \beta$), has area

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Example. Circle:

$$C_R = \left\{ (x, y) \quad | \quad x^2 + y^2 = R^2 \right\}$$

or

$$C_R = \left\{ (r, \theta) \quad | \quad r = R, \quad 0 \leq \theta \leq 2\pi \right\}.$$