Lecture 14

Lecturer: Prof. Sergei Fedotov

10131 - Calculus and Vectors

Techniques of Integration

Lecture 14

1. The area enclosed by the ellipse
2. Hyperbolic substitutions
3. Areas between curves

The area enclosed by the ellipse

Example: Find the area enclosed by the ellipse

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Solving the equation for y, we obtain

$$y = \frac{b}{a}\sqrt{a^2 - x^2}$$

Then

$$\frac{A}{4} = \frac{b}{a}\int_0^a \sqrt{a^2 - x^2}dx$$

To find this integral, we use the substitution: $x = a\sin\theta$....

Hyperbolic substitutions

Example: Find

$$\int \frac{dx}{\sqrt{x^2 + a^2}}.$$

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and the identity:

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Why hyperbolic?

Areas between curves (Problem Sheet 8, 3)

The area A of the region bounded by the curves $y = f(x)$ and $y = g(x)$, and two lines $x = a$, $x = b$, where f and g are continuous and $g(x) \leq f(x)$ for $x \in [a, b]$ is

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Example: Find the area of the region enclosed by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$.

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