Lecture 11

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10131 - Calculus and Vectors

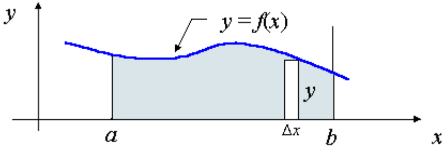
Integrals

Lecture 11

1. The area problem
2. Definition of a definite integral
3. The fundamental theorem of calculus

The area problem

Let us find the area under the graph of $y = f(x)$ between $x = a$ and $x = b$:



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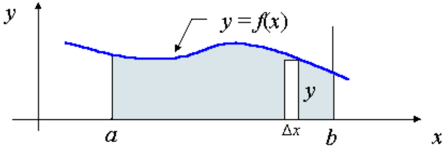

In the graph, a "typical rectangle" is shown with width Δx and height y. Its area is $y\Delta x$. If we add all these typical rectangles, starting from a and finishing at b, the area is approximately:

$$\sum_{i=1}^{n} f(x_i)\Delta x$$

Now if we let$\Delta x \to 0$, we can find the exact area.

Definition of a definite integral

Let f be a function which is continuous on the closed interval $[a, b]$. We divide this interval into n subintervals of equal width $\Delta x = (b - a)/n$.

Let x_i be the endpoints of these subintervals and x_i^* be a sample point in the ith subinterval $[x_{i-1}, x_i]$.

The definite integral of f from a to b is defined as

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{\infty} f(x_i^*)\Delta x$$

provided that this limit exists. If this limit exists, we say that f is integrable on interval $[a, b]$.

The Fundamental Theorem of Calculus

First Fundamental Theorem of Calculus.

Let $f(x)$ be a continuous real-valued function defined on a closed interval $[a, b]$. Let g be the function defined, for all x in $[a, b]$, by

$$g(x) = \int_a^x f(z)dz.$$

Then g is continuous on$[a, b]$, differentiable on the open interval (a, b), and $g'(x) = f(x)$.

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Second Fundamental Theorem of Calculus

Let $f(x)$ be a continuous real-valued function defined on a closed interval $[a, b]$, then

$$\int_a^b f(x)dx = F(b) - F(a),$$

where F is any antiderivative of f, that is $F'(x) = f(x)$. This theorem is employed to compute the definite integral of a function $f(x)$ for which an antiderivative F is known.

Examples

Evaluate the following definite integrals

$$\int_1^2 \frac{1}{x} dx \qquad \int_{-2}^2 \frac{1}{x^2} dx$$