Lecture 10

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10131 - Calculus and Vectors

Euler's formula

1. Euler's formula
2. Polar form of complex numbers

Euler's formula

Show that the Maclaurin series for $\sin x$ is

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

for all $x \in \mathbb{R}$

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The Maclaurin series for $\cos x$:

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$$

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Euler formula: Let us use the Maclaurin series expansion for e^{ix}

$$e^{ix} = 1 + ix + \frac{(ix)^2}{2!} + \frac{(ix)^3}{3!} + \frac{(ix)^4}{4!} + \frac{(ix)^5}{5!} + \ldots = 1 + ix - \frac{x^2}{2!} - \frac{ix^3}{3!} + \frac{x^4}{4!} + \frac{ix^5}{5!}$$

Then....

Polar form of complex numbers

Any complex number $z = a + bi$ can be represented by polar coordinates (r, θ) as $z = a + bi = r\cos\theta + r\sin\theta i$. Thus we can write

$$z = r(\cos\theta + i\sin\theta),$$

where $r = |z| = \sqrt{a^2 + b^2}$ and $\tan\theta = \frac{b}{a}$. The angle θ is called the argument of z. By using Euler's formula we can write

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Example: Write $z = 1 + i$ and $z = \sqrt{3} - i$ in the polar form and $re^{i\theta}$

Taylor series

Example: Find the first two nonzero terms in the Taylor series for

$$f(x) = \sin x$$

at $a = \pi$