Lecture 1

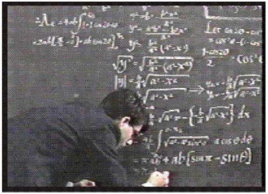
Lecturer: Prof. Sergei Fedotov

10131 - Calculus and Vectors

Sets, intervals, complex numbers

Plan de la présentation

1. Introduction
2. Sets and Intervals
3. Complex numbers



General Information

This course is an introduction to basic aspects of functions, limits, differentiation, etc. Calculus is a large field, and I will not be able to cover all materials. The only way to learn it is to spend time working problems. I would advise you to spend at least 2 hours outside of class for 1 hour in class. Of course, you are no longer at school. You cannot be taught everything in class. It is your responsibility to study.

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The problem sheet for any week is made available via the Sergei Fedotov's homepage: http://www.maths.manchester.ac.uk/sf/.

You should print it out and work through it. Sample answers will be made available on the web later. You will have a small-group supervision class each week and you must work through the problem sheet for that week and hand in your answers (or attempted answers) to the starred questions before your calculus supervision.

Set Notation

A set is a collection of objects, and these objects are called the elements of the set. Notation: $\mathbb{S} = \{2, 4, 6, 8, ...\}$; the set $\mathbb{S}$ consists of all positive even numbers. $6 \in \mathbb{S}$ means that 6 is an element of $\mathbb{S}$.

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Absolute value of a number x, denoted by $\mid x \mid$, is given by

$$\mid x \mid = \begin{cases} x & x \geqslant 0 \\ -x & x < 0 \end{cases}.$$

Examples

For positive number a, $|\, x \,| < a$ if and only if $-a < x < a$.

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Examples:

Solve $\mid x + 2 \mid < 5$.

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Examples:

Solve $| x + 2 |< 5$.

Solve $| x^2 - 1 |< 3$.

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Let us find other powers of i : $i^3 = i^2 i = -i$ and $i^4 = i^2 i^2 = (-1)(-1) = 1$.

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$$z = a + bi,$$

where a is the real part of c and b its imaginary part. This is abbreviated to $a =$Re z and $b =$ Im z.

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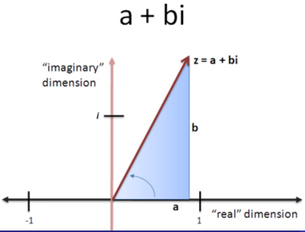
Geometrically, a complex number can be represented as a point on xy plane having the coordinate (a, b).



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Solution: $(3 + i)(1 + 2i) = 3 + 6i + i + i(2i) = 3 + 7i - 2 = 1 + 7i.$

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