# Infrastructure, Public Education and Growth with Congestion Costs

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#### Abstract

This paper studies the optimal allocation of public expenditure between infrastructure and education services in an endogenous growth framework. Raw labor must be educated to become productive. The balanced-growth path is derived and the transitional dynamics associated with an increase in the share of spending on infrastructure are characterized. The growth-maximizing share is shown to depend on the elasticities of output with respect to both infrastructure services and the supply of educated labor. If the supply of raw labor is increasing in wages, the growth-maximizing share of government spending on infrastructure depends negatively on the degree of congestion in schooling.

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## 1 Introduction

Endogenous growth theories have stressed the importance of human capital accumulation as a determinant of growth in per capita income. In a seminal contribution, Lucas (1988) developed a model in which individual decisions to invest in education lead to an increase in the economy's stock of human capital and capacity to produce. The process driving human capital accumulation depends on its current stock and the fraction of non-leisure time that workers devote to training and learning, as opposed to current production. If the returns to education do not decline over time, private spending on education (or investment in human capital) becomes the main source of long-run growth.

A key feature of the Lucas model is that the decision to invest in education, and thus the path of human capital, depends on the individual's decision regarding how much training he or she is willing to undertake. Because everything else in the model depends on the path of human capital, the dynamic behavior of the economy and the steady-state growth rate are completely determined by the way individuals decide to allocate their time. However, as pointed out for instance by Creedy and Gemmell (2002), the hypothesis that education decisions are entirely private ignores the fact that in many developing countries education is provided free of charge, at least at the primary and secondary levels, by the government, and that school attendance is mandatory. Individuals can therefore choose the *intensity* (or level of effort) provided in acquiring education, but the amount of time that is allocated to studying is subject to a lower bound, fixed by government fiat.<sup>1</sup> Moreover, in the presence of credit market imperfections and human capital externalities, private agents may have only weak incentives, and insufficient resources, to

<sup>&</sup>lt;sup>1</sup>By itself, this does not invalidate the Lucas model; the fraction of time allocated to studying can be reinterpreted as the additional time that individuals allocate to homework. In the model of Fisher and Keuschnigg (2002), for instance, self-study (or homework) and school attendance are substitutable inputs (to some degree) in the acquisition of skills.

finance their own education. In such conditions, publicly-provided education can reduce or eliminate the negative externalities that affect individual decisions to accumulate human capital.

Various contributions have extended the Lucas framework to account for government spending on education, as well as other services (such as health, infrastructure, or utility-enhancing services). They include Ni and Wang (1994), Glomm and Ravikumar (1992, 1998, 2003), Baier and Glomm (2001), van Zon and Nuvsken (2001), Fisher and Keuschnigg (2002), Rioja and Glomm (2003), Rivas (2003), and Blankenau and Simpson (2004). Some of these contributions have explicitly studied the extent to which an increase in the public provision of education services raises long-run growth, by altering the process of human capital accumulation. For instance, in Glomm and Ravikumar (1992, 1998, 2003), the learning technology has two inputs: the time that each individual spends studying, and the quality of schools, which is a publicly-provided input common to all individuals. School quality depends on government expenditure, so labor productivity varies with increases in public spending. Blankenau and Simpson (2004) developed an overlappinggenerations model where human capital accumulation results from the provision of both public and private services, which are imperfect substitutes. Public services are proportional to output, whereas the per unit cost of private investment in human capital is proportional to the wage rate. They found that growth depends on the share of government spending on education in output, the ratio of physical capital to human capital, and per capita private investment in human capital. Both sets of studies, however, abstract from the provision of infrastructure and do not consider trade-offs that may arise in the allocation of public expenditure.

This paper departs from the existing literature in several ways. First, it abstracts entirely from private decisions to acquire skills and assumes instead that education is public and free of charge. The absence of an investment function in education for individuals is the consequence of education being mandatory. These assumptions are particularly relevant for low-income developing countries, where the scarcity of human capital have led states to pursue active policies to promote education, and private schooling opportunities are limited. Education, however, is not a pure public good: although it is non-excludable, I assume that it is initially rival as a result of a governmentimposed "admissions" policy that limits the number of individuals who are allowed in schools. Second, I account simultaneously for the provision of education and infrastructure services, in order to study potential trade-offs associated with the allocation of public spending.<sup>2</sup> The growth effects of public spending on infrastructure have attracted much interest in recent years. In an early contribution by Barro (1990), public investment was treated as a flow; subsequent contributions by Turnovsky and Fisher (1995), Chang (1999), Fiaschi (1999), Turnovsky (2000), and Eicher and Turnovsky (2000), followed that approach as well. In the model developed in this paper, growth depends also on the flow of government spending on infrastructure, in addition to the provision of education services. As a result, the optimal allocation of tax revenue can be examined. The fact that all components of spending are productive, and that the government faces a budget constraint, makes this issue non trivial from a growth perspective—particularly for low-income countries, where needs are great in both education and infrastructure.

Third, the model assumes that the economy is endowed only with "raw" labor, and that raw labor must be educated to become productive. Knowledge is thus "embodied" in workers, unlike Lucas-type models where human capital is disembodied and can therefore grow without bounds. Fourth, the model accounts for congestion costs in education—an important feature of education systems in the developing world, particularly in low-income coun-

<sup>&</sup>lt;sup>2</sup>Rivas (2003) for instance examined the impact of changes in the allocation of government spending (for given tax rates) between government consumption, transfer payments, and the provision of infrastructure services in an endogenous growth framework. He did not, however, account for human capital accumulation and publicly-provided education services, and did not derive optimal allocation rules.

tries. According to World Bank estimates, in 1999, the pupil-teacher ratio in primary schooling (a common indicator of the quality of education) was 16.9 for high-income countries, but reached 21.4 in middle-income countries and 38.9 in low-income countries. In the same year, the ratio was 41.5 in South Asia and 46.7 in sub-Saharan Africa. Most recent estimates for those regions put the ratio at 40 and 44, respectively (see UNESCO (2005)). Overcrowded classrooms affect the benefits of public education, both in terms of quality and quantity. Infrastructure-related congestion costs have been studied in several contributions in the endogenous growth literature,<sup>3</sup> but congestion costs (or quality issues) associated with the provision of education services have not, as far as I know, been dealt with in detail. Glomm and Ravikumar (1992, 1998, 2003) relate the quality of schools to public spending on education (as noted earlier) but, given their assumption of a linear relationship between these two variables, they do not allow for congestion effects. They also abstract from infrastructure spending. The only contribution that I am aware of is a study by Tamura (2001). He allows for congestion effects by introducing a trade-off between teacher quality and class size in the production of human capital in determining school quality (that is, smaller classes provide better learning environments, but the detrimental effects of larger classes can be mitigated by improving the quality of teachers). He focuses, however, on convergence issues, rather than the optimal allocation of public resources as I do here.<sup>4</sup>

The remainder of the paper is organized as follows. Section II presents the basic framework. Section III discusses the balanced-growth equilibrium and the dynamic properties of the model. Section IV examines the shortand long-run effects of an increase in the share of government spending on

 $<sup>^{3}</sup>$ See for instance Fisher and Turnovsky (1998), Glomm and Ravikumar (1999), and Eicher and Turnovsky (2000), where the use of public capital is congested by the use of private capital.

<sup>&</sup>lt;sup>4</sup>Rioja and Glomm (2003) consider both public education and public provision of infrastructure services, as I do here. However, they do not provide an explicit analysis of the optimal allocation of public resources.

infrastructure services. Section V derives the growth-maximizing allocation of public expenditure between education and infrastructure. Section VI introduces congestion costs in public education. The last section of the paper summarizes the main results.

## 2 The Economy

Consider an economy populated by a single infinitely-lived household who produces and consumes a single traded good, which can be used for consumption or investment. The economy's endowment consists of raw labor, which must be educated to be used in the production process. The government provides infrastructure and education services (with the former consisting of spending on transportation, communication, sewers, water systems, and so on, and the second consisting of expenditure on books, lunches, and so on) free of charge. It levies a flat tax on output to finance its outlays. All individuals in the raw labor force (which grows at a constant rate) seek to acquire skills; however, not all of them have access to the education system. The number of students is set through an "admissions" policy, which involves (as discussed later) some form of rationing. As a result, whereas infrastructure services are a pure (that is, non-rival, non-excludable) public good, education is not; it is non-excludable (an uneducated individual cannot prevent other individuals from accessing the education services that the government provides at no cost), but it is rival (the use of the education system by a sufficient number of uneducated individuals precludes its use by others).

#### 2.1 Production

Output, Y, is produced with private physical capital, public infrastructure services, and educated labor, using a Cobb-Douglas technology:

$$Y = G^{\alpha} E^{\beta} K_P^{1-\alpha-\beta},\tag{1}$$

where  $K_P$  is the stock of private capital, G government services, E the stock of educated labor, and  $\alpha, \beta \in (0, 1)$ . Thus, production exhibits constant returns to scale in all factors. Moreover, as long as  $G/K_P$  and  $E/K_P$  are constant, output is proportional to the private capital stock; the production function is then an AK-type technology, which implies that the equilibrium is characterized by steady-state growth.

#### 2.2 Household Preferences

Assuming no disutility associated with working, and no utility *per se* from the acquisition of skills, the infinitely-lived household maximizes the discounted stream of future utility<sup>5</sup>

$$\max_{C} V = \int_{0}^{\infty} \ln C_t \exp(-\rho t) dt, \qquad (2)$$

where C is consumption and  $\rho > 0$  the discount rate. Consumption enters the instantaneous utility function in logarithmic form, implying that income and substitution effects cancel out, and that the household's propensity to save (and invest) is independent of the rate of return on capital. Moreover, unlike some contributions in the literature—such as Barro (1990), Turnovsky and Fisher (1995), Baier and Glomm (2001), Rioja and Glomm (2003), and Turnovsky (2004)—I do not allow for utility-enhancing public services.<sup>6</sup>

The household budget constraint is

$$C + \dot{K}_P = (1 - \tau)Y,\tag{3}$$

where  $\tau \in (0, 1)$  is the tax rate on output. For simplicity, the depreciation rate of private capital is assumed to be zero.

<sup>&</sup>lt;sup>5</sup>Throughout the paper, the time subscript t is omitted whenever doing so does not result in confusion. A dot over a variable is used to denote its time derivative.

<sup>&</sup>lt;sup>6</sup>For a more general specification of instantaneous utility in this class of models, see Agénor (2005b, 2005d).

Maximizing (2) subject to (1) and (3) yields

$$\frac{\dot{C}}{C} = (1 - \alpha - \beta)(1 - \tau)(\frac{G}{K_P})^{\alpha}(\frac{E}{K_P})^{\beta} - \rho, \qquad (4)$$

together with the transversality condition  $\lim_{t\to\infty} (K_P/C) \exp(-\rho t) = 0.$ 

#### 2.3 Human Capital Accumulation

The schooling technology is specified as a two-level production function. At the first level, the prevailing quantity of educated labor, E, and government spending on education,  $I_E$ , are combined to produce a composite input. At the second level, this input is combined with the number of individuals seeking to acquire an education, L, to produce the flow of newly-educated workers,  $\dot{E}$ . Thus, a more literate environment leads to the production of a greater number of educated workers, for given levels of public spending on education and individuals seeking to acquire skills.<sup>7</sup>

Assuming that technology is Cobb-Douglas at both levels yields:

$$\dot{E} = A (I_E^{\omega} E^{1-\omega})^{\eta} L^{1-\eta}, \qquad (5)$$

where A is a scale parameter and  $\omega, \eta \in (0, 1)$ . The schooling technology exhibits therefore constant returns to scale in E and  $I_E$  (taken separately), as well as in the composite input  $I_E^{\omega} E^{1-\omega}$  and L.

Equation (5) can be rewritten as

$$\dot{E} = A\left(\frac{I_E^{\omega}E^{1-\omega}}{L}\right)^{\eta}L = A\left(\frac{I_E}{E}\right)^{\omega\eta}\left(\frac{E}{L}\right)^{\eta}L.$$
(6)

As shown later, in the steady state the growth rate of educated labor is positive. However, given the schooling technology (5), E cannot grow

<sup>&</sup>lt;sup>7</sup>Note that it could have been assumed (as in Agénor (2005*b*, 2005*c*)) that a fraction  $\chi$  of the total stock of educated labor consists of teachers on the government's payroll, with the rest engaged in the production of goods. However, this would only change the definitions of the constant terms in (1) and (5) and would not affect the results in any way, as long as  $\chi$  is constant.

without bound; it cannot, in fact, exceed the growth rate of the stock of raw labor (students) admitted in schools,  $n.^8$  Thus,  $\dot{E}/E \leq n$ . In order to abstract from considerations related to endogenous population growth, I impose this restriction in a particularly simple way—I assume a strictly proportional relation between E and L, of the form  $L = \varphi E$ , where  $\varphi > 0$ . Thus, in the steady state, L grows at the same rate as E, implying that the ratio L/E is constant. In effect, this restriction amounts to assuming that the government rations access to public education by fixing n. The case where L responds endogenously to changes in wages is examined later.<sup>9</sup>

Substituting  $L = \varphi E$  in (6) implies that the growth rate of E evolves over time according to

$$\frac{\dot{E}}{E} = B(\frac{I_E}{E})^{\omega\eta},\tag{7}$$

where  $B = A\varphi^{1-\eta}$ . For simplicity, there is no "depreciation" of educated labor.<sup>10</sup>

#### 2.4 Government

To finance the provision of infrastructure and education services, the government collects a proportional tax on output at the rate  $\tau \in (0, 1)$ .<sup>11</sup> Thus, the government budget constraint is given by

$$I_E + G = \tau Y. \tag{8}$$

<sup>&</sup>lt;sup>8</sup>Note that n is endogenous, whereas the growth rate of the overall raw labor force itself is exogenous and bounded from below by n.

<sup>&</sup>lt;sup>9</sup>Note also that if it had been assumed (as indicated earlier) that a fraction of the stock of educated labor consists of public sector teachers, the interpretation of this restriction would be straightforward: it would mean that the government is trying to achieve a constant pupils-to-teachers ratio by limiting the number of individuals accepted in the classrooms.

<sup>&</sup>lt;sup>10</sup>Note also that I have not accounted for the possibility that public infrastructure may affect the schooling technology and therefore the ability to produce educated labor; this issue is discussed at length elsewhere (see Agénor (2005a, 2005b)).

<sup>&</sup>lt;sup>11</sup>The working paper version of this article considers the case where, as in Park and Philippopoulos (2004), the government collects a proportional tax on installed capital, as well as the case of lump-sum taxation.

Assuming that infrastructure services are a constant fraction of tax revenue, so that  $G = v\tau Y$ , with  $v \in (0, 1)$ , the government budget constraint can be rewritten as

$$I_E = (1 - v)\tau Y. \tag{9}$$

### 3 The Balanced-Growth Equilibrium

The derivation of the balanced-growth equilibrium (BGE) is provided in the working paper version of this article (see Agénor (2005*a*)). There it is shown that the model can be condensed into a system of two nonlinear differential equations in  $c = C/K_P$  and  $e = E/K_P$ . This system, together with the initial condition  $e_0 > 0$ , and the transversality condition  $\lim_{t\to\infty} c^{-1} \exp(-\rho t) = 0$ , characterize the dynamic equilibrium. The BGE is a set of functions  $\{c, e\}_{t=0}^{\infty}$  such that the dynamic equations and the transversality condition are satisfied, and consumption, the stock of educated labor, and the stock of private capital, all grow at the same constant rate  $\gamma$ .<sup>12</sup> This rate (which is also the rate of growth of output) is given by the equivalent forms

$$\gamma = (1 - \alpha - \beta)(1 - \tau)(\tau \upsilon)^{\alpha/(1 - \alpha)} \tilde{e}^{\beta/(1 - \alpha)} - \rho, \qquad (10)$$

$$\gamma = B[\tau(1-\upsilon)(\tau\upsilon)^{\alpha/(1-\alpha)}]^{\omega\eta}\tilde{e}^{-\chi}, \qquad (11)$$

where  $\chi \equiv (1 - \alpha - \beta)\omega\eta/(1 - \alpha) > 0$ , and a"~" is used to characterize a steady-state value. In the working paper version, it is also shown that the dynamic system in c and e is saddlepath stable and that the equilibrium is unique. The model is thus locally determinate.

The phase diagram in Figure 1 shows how the BGE equilibrium is reached. The phase curve CC represents the combinations of c and e for which the consumption-capital stock ratio is constant ( $\dot{c} = 0$ ), whereas the phase curve

<sup>&</sup>lt;sup>12</sup>The transversality condition is satisfied along any interior BGE because consumption and the stock of private capital grow at the same constant rate, implying that the ratio  $c = C/K_P$  is also constant.

EE represents the combinations of c and e for which the educated laborcapital stock ratio is constant ( $\dot{e} = 0$ ). Both curves have a concave shape, but saddlepath stability requires that the slope of EE be steeper than the slope of CC at the point at which they intersect, point A, which corresponds to the BGE. The saddlepath, denoted SS in Figure 1, has a positive slope.

## 4 Increase in the Share of Infrastructure

Consider now an unanticipated, permanent increase in the share of spending on infrastructure, v, for a constant tax rate,  $\tau$ . Given the balanced-budget assumption (see (8)), an increase in v leads to a concomitant reduction in the share of spending on education services. Because both types of services affect production (directly, in the case of infrastructure, indirectly, in the case of education), it is intuitively clear that this policy entails a trade-off with respect to its impact on the economy's growth rate. Indeed, in the long run, an increase in the share of spending on infrastructure services leads to a lower ratio of educated labor to physical capital in the long run. By contrast, as established in detail in the working paper version, whether the steady-state consumption-capital ratio and the balanced growth rate increase depends on the ratio  $\alpha/\beta$ , that is, the relative elasticities of output with respect to infrastructure services and educated labor (see Agénor (2005a)). The higher this ratio relative to the elasticity of the steady-state value of the educatedlabor capital ratio with respect to v,  $\varepsilon_{\tilde{e}/v}$ , the more likely it is that the consumption-capital ratio and the growth rate will increase.

The transitional dynamics associated with an increase in v are illustrated in Figure 2, for  $\alpha/\beta$  higher and lower than  $\varepsilon_{\tilde{e}/v}$ . A rise in v leads to a leftward shift in both CC and EE. In the upper panel of the figure,  $\alpha/\beta > \varepsilon_{\tilde{e}/v}$ , and the consumption-capital ratio jumps up on impact from A to B, located on the new saddlepath S'S'. This leads to a reduction in the ratio of educated labor to private capital ( $\dot{e}_0 < 0$ ). Over time, both c and e fall along S'S'. In the lower panel of the figure,  $\alpha/\beta < \varepsilon_{\tilde{e}/v}$ , and the consumption-capital ratio jumps downward on impact from A to B, and continues to decline (together with e) along S'S' during the transition. In both cases, the economy converges monotonically to the new BGE, located at point A'.

## 5 The Growth-Maximizing Policy

The foregoing discussion implies that there is a hump-shaped curve linking the growth rate and v, and thus a growth-maximizing value for that variable, similar to that obtained by Barro (1990) in a setting where the tax rate on output and the share of spending on infrastructure are one and the same.<sup>13</sup>

From (10) and (11), the growth-maximizing share of spending on infrastructure can be derived as

$$v^* = \frac{\alpha}{\alpha + \beta}.\tag{12}$$

Thus, an increase in  $\alpha$  raises  $v^*$ , whereas an increase in  $\beta$  lowers  $v^*$ . If  $\beta = 0$ , so that educated labor has no effect on private production, the optimal share of spending on infrastructure is unity. Conversely, if  $\alpha = 0$  (that is, spending on infrastructure services has no effect on private output), then  $v^* = 0$ . Note also that the optimal share  $v^*$  is independent of the education technology, as captured by the parameters  $\omega$  and  $\eta$ .

## 6 Congestion Costs

To introduce congestion costs in the present setting, I assume that government spending on education,  $I_E$ , is less productive the higher the number of

<sup>&</sup>lt;sup>13</sup>See Tsoukis and Miller (2003), and Zagler and Durnecker (2003) for a review of Barro's results and its extensions. In Barro's (1990) model, with output taxes, the growth rate declines after a point with increases in the tax rate, as the adverse impact of distorting taxes (on private savings and investment) dominates the positive effect of public spending on the marginal productivity of capital. See Appendices A and B of the working paper version of this article for a derivation of the optimal tax rate in the present setting, and Agénor (2005b) for a more detailed discussion.

individuals in the raw labor force seeking to acquire skills. For instance, if  $I_E$  represents spending on books used in the classroom, a greater number of students means that books must be shared, thereby making learning more laborious. Equation (5) is then replaced by

$$N = A[(\frac{I_E}{L^{\phi}})^{\omega} E^{1-\omega}]^{\eta} L^{1-\eta} = A(\frac{I_E}{E})^{\omega\eta} E^{\eta} L^{1-\eta-\phi\omega\eta}, \qquad (13)$$

where  $\phi \in (0, 1)$  measures the degree of congestion. Thus, an increase in the economy's stock of raw labor seeking an education reduces the efficiency of the education system and lowers the flow supply of educated labor if  $1 - \eta - \phi \omega \eta < 0$ , that is,  $\phi > (1 - \eta)/\omega \eta$ . If  $\phi = 1$ , then it is the provision of education services per school attendant,  $I_E/L$ , which determines the quantity of educated labor produced at any moment in time, as in Beauchemin (2001, p. 294) for instance. In the logic of Tamura (2001),  $I_E/L$  can then also be interpreted as an indicator of the quality of schooling.

To focus attention on the issue at hand (the impact of congestion costs on the optimal allocation rule), and to simplify algebraic results, I assume that  $\eta = 1$  and  $\delta_E = 0$ , so that  $\dot{E} = N$ . Thus, equation (13) yields

$$\frac{\dot{E}}{E} = A(\frac{I_E}{E})^{\omega} L^{-\theta},\tag{14}$$

where  $\theta = \phi \omega$ . Thus, an increase in L unambiguously lowers the growth rate of the stock of educated labor.

Suppose also that now the number of uneducated individuals seeking to acquire skills depends positively on the going wage paid to educated labor.<sup>14</sup> Implicitly, therefore, the pay-off to remaining uneducated is zero (or, more generally, constant). In the present setting, with continuous clearing of the

<sup>&</sup>lt;sup>14</sup>In principle, of course, it is the discounted present value of all future wages that should affect schooling decisions. However, this would complicate quite significantly the model and make the derivation of explicitly analytical solutions very difficult. Despite its relatively *ad hoc* nature, the specification chosen here is sufficient to illustrate the impact of congestion in education on the optimal share of spending on infrastructure.

labor market, this wage is equal to the marginal product of educated labor, which from (1) is given by  $\beta G^{\alpha} E^{\beta-1} K_P^{1-\alpha-\beta}$ . Thus,

$$L = \Gamma \left[ \beta \left( \frac{G}{K_P} \right)^{\alpha} \left( \frac{K_P}{E} \right)^{1-\beta} \right], \tag{15}$$

where, in general,  $\Gamma' > 0$  and  $\Gamma'' < 0$ . The raw labor supply decision (or, more precisely, the supply of raw labor seeking to acquire skills through public schools) is thus assumed to be separable from consumption decisions.<sup>15</sup>

Noting that  $G/K_P = (\tau \upsilon)^{1/(1-\alpha)} e^{\beta/(1-\alpha)}$ , and taking a linear approximation to  $\Gamma$ , yields

$$L = \beta \Gamma'(\tau \upsilon)^{\alpha/(1-\alpha)} e^{-(1-\alpha-\beta)/(1-\alpha)}.$$
(16)

Substituting (16) in (14) yields

$$\frac{\dot{E}}{E} = A[\beta\Gamma'(\tau\upsilon)^{\alpha/(1-\alpha)}]^{-\theta}[(\frac{I_E}{K_P})(\frac{K_P}{E})]^{\omega}e^{\theta(1-\alpha-\beta)/(1-\alpha)}$$

which, noting that  $I_E/K_P = \tau (1-\upsilon)(\tau \upsilon)^{\alpha/(1-\alpha)} e^{\beta/(1-\alpha)}$ , can be rearranged to give

$$\frac{\dot{E}}{E} = \frac{A[\tau(1-\upsilon)(\tau\upsilon)^{\alpha/(1-\alpha)}]^{\omega}}{[\beta\Gamma'(\tau\upsilon)^{\alpha/(1-\alpha)}]^{\theta}}e^{-\chi'},\tag{17}$$

where  $\chi' \equiv (1 - \alpha - \beta)(\omega - \theta)/(1 - \alpha) > 0.$ 

Using now (17), the dynamic system driving the economy can also be written in terms of c and e; this system is also saddlepath stable.

The steady-state growth rate is now given by the equivalent forms (10), and, given (17),

$$\gamma = \frac{A[\tau(1-\upsilon)(\tau\upsilon)^{\alpha/(1-\alpha)}]^{\omega}}{[\beta\Gamma'(\upsilon\tau)^{\alpha/(1-\alpha)}]^{\theta}}\tilde{e}^{-\chi'}$$

It can be established that the optimal share is now

$$v^* = \frac{\alpha(\omega - \theta)}{\alpha(\omega - \theta) + \omega\beta} = \frac{\alpha(1 - \phi)}{\alpha(1 - \phi) + \beta}.$$
 (18)

<sup>&</sup>lt;sup>15</sup>More formally, raw labor supply could be assumed to enter separately in the instantaneous utility function, as for instance in Greiner (1999) or Palivos, Yip, and Zhang (2003), and solved for as part of the household's optimization problem. Given that this does not add much insight to the issue at hand, I restrict the discussion to specification (15).

This formula is simply (12) if  $\phi = 0$ . It implies that the higher the degree of congestion in education, the *lower* the optimal share of government spending on infrastructure services  $(dv^*/d\phi < 0)$ . The reason is as follows. An increase in v, by raising the marginal product of educated labor, brings more students into public schools. Increased congestion tends to lower the supply of educated labor and thus the educated labor-capital ratio, which further increases the supply of raw labor (as a result of decreasing marginal returns to educated labor in production). The optimal policy, in response to higher congestion effects, is to increase spending on education (or, equivalently, reduce spending on infrastructure services), in order to offset this adverse effect on growth. With full or "proportional" congestion, that is, with  $\phi = 1$ , the optimal share of spending on infrastructure is zero. For instance, with  $\alpha = 0.15$ ,  $\beta = 0.45$  (as in Agénor (2005*c*), for instance) the optimal share of spending on infrastructure is 25 percent of tax revenues with  $\phi = 0$ , but only 14 percent with  $\phi = 0.5$ .

As one would expect, these results depend crucially on the way the inflow of raw labor into public schools is modeled. Suppose, for instance, that the smaller the average quantity of physical capital that educated individuals have access to, the lower the incentive to acquire skills. Unlike the specification in (16), the supply of individuals seeking an education would be *positively* related to the ratio of educated labor to physical capital, e, perhaps because these individuals value the fact that the use of machines increases learning opportunities (as in Kosempel (2004)) and improves their productivity. This assumption can be captured in a simple (albeit *ad hoc*) manner by setting  $L = e^{\kappa}$ , with  $\kappa > 0$ . As a result, and using (14), equation (17) is replaced by

$$\frac{\dot{E}}{E} = A[\tau(1-\upsilon)(\tau\upsilon)^{\alpha/(1-\alpha)}]^{\omega}e^{-\eta''},$$

where  $\chi'' \equiv \omega (1 - \alpha - \beta) / (1 - \alpha) + \kappa \theta > 0.$ 

Following the same reasoning as before, it can now be established that

$$v^* = \frac{\alpha(1 + \kappa\phi)}{\alpha(1 + \kappa\phi) + \beta}.$$
(19)

This result is identical to (12) if either  $\kappa = 0$  or  $\phi = 0$ . But now an increase in congestion costs *raises* the optimal share of spending on infrastructure services  $(dv^*/d\phi > 0)$ . The reason why the adverse effect of a rise in  $\phi$  on the steady-state growth rate can be mitigated by a higher v is because an increase in v lowers the (steady-state) educated labor-capital ratio, as established earlier, it now reduces the number of individuals seeking to acquire skills. The adverse effect of a greater degree of congestion in public schools on growth can therefore be offset by spending less on education and more on infrastructure.

## 7 Summary

This paper studied the determination of the optimal allocation of public resources between infrastructure and education services, in a model where raw labor must be educated to become productive. The growth-maximizing share of public spending on infrastructure was shown to depend only on the parameters characterizing the production technology. The model was then extended to account for congestion effects in education—an issue that has received scant attention in the literature, despite the importance of these effects in developing countries. It was shown that, depending on what determines the incentive to seek education in public schools, the growth-maximizing share may depend either positively or negatively on the degree of congestion in education. In particular, even if the number of pupils has only negative effects on the rate of human capital accumulation, this does not imply that spending on education (infrastructure) should be reduced (increased) in response to an increase in the degree of congestion in schooling; on the contrary, if the decision to acquire skills is a function of the current wage (viewed perhaps as a proxy for future wages), the optimal response is a reduction in the share of spending on infrastructure.

The model developed in this paper can be extended in a variety of directions. One extension would be to account for private education (and subsidies to private schools), in addition to public education. This would allow an analysis of the growth and distributional effects of the two regimes, as in Zhang (1996), Glomm and Ravikumar (1992, 2003), and Glomm and Kaganovich (2003), in the presence of trade-offs between public education and infrastructure spending.

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Figure 1 The Steady-Growth Equilibrium



# Figure 2



