CYCLIC GROUP ACTIONS ON POLYNOMIAL RINGS

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Abstract. We consider a cyclic group of order p^n acting on a module incharacteristic p and show how to reduce the calculation of the symmetric algebra to that of the exterior algebra.

Consider a cyclic group of order p^n acting on a polynomial ring $S = k[x_1, \dots, x_r]$, where k is a field of characteristic p; this is equivalent to the symmetric algebra $S^*(V)$ on the module V generated by $x_1, \dots, x_r$. We would like to know the decomposition of S into indecomposables.

This was calculated by Almkvist and Fossum in [1] in the case$n = 1$; see also [6]. They reduced the problem to the calculation of the exterior powers of V, and then gave a formula for these.

In this note we accomplish the first part for general n, that is to say the reduction of the calculation of the symmetric algebra to that of the exterior algebra. Many of the results extend to a group with normal cyclic Sylow p-subgroup, in particular to any finite cyclic group.

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1. Periodicity

Let G be a finite group, k a field of characteristic p and V a kG-module. Suppose that $V' < V$ is a kG-submodule of codimension 1, and let $x \in V \setminus V'$. Set $a = \Pi_{g \in G/\operatorname{Stab}_G(x)} gx^r$, for some $r \in \mathbb{N}$. Choose a basis for V' and add x to obtain a basis of V. Let $T \leq S = S^*(V)$ be the kG-submodule spanned by the monomials in the basis elements that are not divisible by $x^{\deg a}$.

Lemma 1.1. *As kG-modules, $S \cong k[a] \otimes_k T$.*

Proof. (cf. [7]) Since V/V' is a simple module we must have $\operatorname{rad} V \leq V'$. Now x is an eigenvector modulo $\operatorname{rad} V$, so $a \equiv \lambda x^{\deg a} \mod T$ for some $0 \neq \lambda \in k$. Since $S = x^{\deg a} S \oplus T$, we obtain $S = aS \oplus T$, and the result follows by repeated substitution for S. □

Remark. It is easy to extend this to a multiple periodicity, with one polynomial generator for each 1-dimensional summand of $V/\operatorname{rad} V$.

Theorem 1.2. *Suppose that G has a normal cyclic Sylow p-subgroup H, and that V is a non-zero kG-module that is indecomposable on restriction to H. Then $S^*(V) \cong k[a] \otimes B$ modulo modules projective relative to proper subgroups of H, where a is an eigenvector for G of degree $|H|$ and B is a sum of homogeneous submodules of degree strictly less than $\deg a$.*

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Remark. This was conjectured in the case $G = H$ by Hughes and Kemper [6].

Proof. There must be a complement F to H in G. This is because the extension $H \to G \to G/H$ is classified by an element of $H^2(G/H, H)$, and this group is 0 since $|G/H|$ is prime to $|H|$, (or invoke the Schur-Zassenhaus Theorem).

Let $\dim V = r$ and $|H| = p^n$. We assume that V is faithful, so $r > p^{n-1}$ and hence $n \leq r - 1$.

Now $L = V/\operatorname{rad} V$ is 1-dimensional ([2] II 5 8), so can be written over a finite field $\mathbb{F}_q$. The map $V \to L$ splits over F, since $|F|$ is not divisible by p: let $x \in V$ be a non-zero element in the image of the splitting. Then V is generated over G by x, and x is an eigenvector for F. We set $a = \Pi_{h \in H} hx$, so a is an eigenvector for G of degree p^n.

Let $W_r(k)$ denote the group of invertible upper-triangular $r \times r$-matrices over k and $U_r(k)$ its subgroup of matrices with entries 1 on the diagonal.

The projective cover of L is $\operatorname{Ind}_F^G L$, so V is a quotient of this. It follows that V can be realized over $\mathbb{F}_q$, and from now on we suppose that $k = \mathbb{F}_q$. It also follows that we may regard G as a subgroup of $W_r(k)$ and V as the restriction to G of the natural representation of $W_r(k)$, with x as top basis element. Let d_r be the orbit product of x under $U_r(k)$, so $\deg d_r = q^{r-1}$.

The previous Lemma 1.1 tells us that $S \cong k[a] \otimes T$ over H, where T is spanned by the monomials in the basis elements that are not divisible by x^{p^n}, and also $S \cong k[d_r] \otimes T'$ over $U_r(k)$, where T' is spanned by the monomials in the basis elements that are not divisible by $x^{q^{r-1}}$. In the first of these isomorphisms, the right hand side is naturally a kG-module, since a is an eigenvector for G. The natural map from right to left is G-equivariant, so the isomorphism is valid over G.

Observe that a generator of H can not be contained in any of the s-row subgroups of $U_r(k)$ for $s \leq r - 2$, i.e. the subgroups $U_J < U_r$ for $J \subsetneqq I = \{1, \ldots, r-1\}$ consisting of elements with zero off-diagonal elements in the rows corresponding to the elements of J. This is because such a group has a fixed space on the dual module V^* of dimension at least 2, and the dimension of the fixed space of H on V^* is 1.

The Main Theorem of [7] as generalized to $W_r(k)$ in [8] 7.1 states that, as a $kW_r(k)$-module, T' is a sum of pieces of the form $D \otimes \bar{X}_J(I) \cong D \otimes \operatorname{Ind}_{W_J}^{W_r} \bar{X}_J(J)$, (where D is 1-dimensional and W_J is defined analogously to U_J). The double coset formula (see e.g. [3] 3.3.4) and the observation above show that, as kG-modules, $T' \cong B'$ modulo summands induced from subgroups with strictly smaller Sylow p-subgroups, where B' is the module $\bar{X}_I(I)$ of that paper, (which only occurs once in the decomposition of T'), so a sum of homogeneous pieces known to be of degree less than $\deg d_r$.

It is easy to check that, over kG, $T' \cong T \oplus aT \oplus \cdots \oplus a^{q^{r-1}p^{-n}-1}T$. In particular T can only have a finite number of summands that are not induced; call their sum B. Then $B' \cong B \oplus aB \oplus \cdots \oplus a^{q^{r-1}p^{-n}-1}B$.

This shows that $\deg B' = \deg a^{q^{r-1}p^{-n}-1} + \deg B$ (where deg denotes the maximum degree of the homogeneous pieces). Thus $\deg B = \deg B' - q^{r-1} + p^n$ and, since $\deg B' < q^{r-1}$, we obtain $\deg B < p^n$. □

Remark. If F is abelian and k is a splitting field then all indecomposable kG-modules satisfy the condition of the theorem (see e.g. [2] II 6).

2. Working modulo induced modules

Let C_{p^n} denote the cyclic group of order p^n and let $A(C_{p^n})$ denote the Green ring of kC_{p^n}-modules, (where k has characteristic p).

Recall that $kC_{p^n} \cong k[X]/X^{p^n}$, where X corresponds to $g-1$ for some generator $g \in C_{p^n}$. The indecomposable representations of C_{p^n} are $V_t = k[X]/X^t$, up to isomorphism, where $1 \le t \le p^n$ (see [2], for example). For convenience we let V_0 denote the 0 module. The decomposition of tensor products is described implicitly in [5] and explicitly in [9].

For $m < n$ there are restriction and induction operators $R^n_m : A(C_{p^n}) \to A(C_{p^m})$ and $I^n_m : A(C_{p^m}) \to A(C_{p^n})$ with the properties that $I^n_m V_t = V_{p^{n-m}t}$, $R^n_m I^n_m = p^{n-m}$ and $I^n_m R^n_m = V_{p^{n-m}} \otimes -$.

For any finite dimensional kC_{p^n}-module V, let $H^n = H^n(V)$ be the element of $A(C_{p^n})[[t]]$ with $S^r(V)$ as coefficient of t^r. Note that $H^n(V \oplus W) = H^n(V)H^n(W)$.

Let $P^n(V)$ be the part of $H^n(V)$ where we only consider indecomposables of dimension prime to p, or, equivalently, those not induced from a proper subgroup. In other words, we write $S^n(V)$ as a sum of indecomposables, omit all the summands of dimension prime to p and put the remainder as the coefficient of t^n in $P^n(V)$.

Lemma 2.1. *If V is indecomposable then $P^n(V) = Q^n(V)(1-t^{p^n})^{-1}$, where $Q^n(V)$ is a polynomial of degree $< p^n$.*

Proof. This is part of the periodicity result 1.2 above. □

Proposition 2.2. *Suppose that we know the $P^m(V)$ for $m \le n$. Then we also know the $H^n(V)$, and, in fact,*

$$H^n = (1-p^{-1}V_p)P^n + p^{-1}I^n_{n-1}((1-p^{-1}V_p)P^{n-1}) + p^{-2}I^n_{n-2}((1-p^{-1}V_p)P^{n-2}) + \cdots .$$

In this formula, P^{n-r} is an abbreviation for $P^{n-r}R^n_{n-r}V$, and V_p is the indecomposable module of dimension p for $C_{p^{n-r}}$.

Proof. Write $R = R^n_{n-1}$ and $I = I^n_{n-1}$. Now $IRI = pI$, so on inducedmodules we have $p^{-1}IR = 1$. Thus $H^n - P^n = p^{-1}IR(H^n - P^n) = p^{-1}(IH^{n-1} - IRP^n) = p^{-1}(IH^{n-1} - V_pP^n)$, and so $H^n = p^{-1}IH^{n-1} + (1-p^{-1}V_p)P^n$.

Now use recursion in n. □

For kG-modules $W \le V$, write (V,W) for the pair. There is anobvious definition of direct sum.

Lemma 2.3. *For a cyclic p-group G and indecomposable G-module V, every $\mathrm{Res}^G_H(V,W)$ is a sum of (V',W') with V' (and W') indecomposable and $\mathrm{codim}(V',W') \ge \lfloor \mathrm{codim}(V,W)/|G:H| \rfloor$.*

Proof. By induction, we may assume that H is of index p in G. So $kG \cong k[X]/X^{p^n}$ and $kH \cong k[X^p]/X^{p^n} \cong k[Y]/Y^{p^{n-1}}$, where $Y = X^p$.

Also we may take $V = k[X]/X^v$ and $W = X^{v-w}k[X]/X^v$, where $v = \dim V$ and $w = \dim W$.

Now $\mathrm{Res}^G_H V = k[Y]/X^v \oplus \cdots \oplus X^{p-1}k[Y]/X^v$and $\mathrm{Res}^G_H W = X^{v-w}k[Y]/X^v \oplus \cdots \oplus X^{v-w+p-1}k[Y]/X^v$.

Write $v-w = pa+b$ with $0 \le b \le p-1$. Then, with a little rearranging, $\mathrm{Res}^G_H W = Y^{a+1}k[Y]/X^v \oplus \cdots \oplus X^{b-1}Y^{a+1}k[Y]/X^v \oplus X^bY^ak[Y]/X^v \oplus \cdots \oplus X^{p-1}Y^ak[Y]/X^v$; each of these summands is clearly a submodule of the summand of $\mathrm{Res}^G_H V$ labeled by the same power of X, and we need to show that the codimension is at least a.

Now $X^rY^sk[Y]/X^v \leq X^rk[Y]/X^v$ has codimension s provided that X^rY^{s-1} is not 0 in the module, which happens when $v-r > p(s-1)$, and in our cases s is equal to a or $a+1$.

We claim that in all our cases in fact $v-r > p(s-1)$. In the cases when $s = a+1$ we would otherwise have $v-r \leq pa = v-w-b$, hence $r \geq b+w$. But in these cases $r \leq b-1$, a contradiction. In the cases when $s = a$ a similar argument gives $r \geq b+w+p$, when we know that $r \leq p-1$. □

3. Symmetric Powers

Let $H < G$, and let B be a G-module projective relative to H. Let $f : A \to B$ be a map of G-modules.

Lemma 3.1. *If f, considered as a map of H-modules, factors through a projective then it also factors through a projective as a map of G-modules. The same claim also holds for any $g : B \to A$.*

Proof. Recall that, for groups $U \leq V \leq G$ and V-modules A, B, we define $\mathrm{tr}^V_U : \mathrm{Hom}_U(A,B) \to \mathrm{Hom}_V(A,B)$ by $\mathrm{tr}^V_U(\phi) = \sum_{v \in V/U} v\phi v^{-1}, \quad \phi \in \mathrm{Hom}_U(A,B)$.

The hypotheses are equivalent to $f = \mathrm{tr}^H_1 f'$ and $\mathrm{Id}_B = \mathrm{tr}^G_H i$, where i is H-equivariant (see e.g. [3] 3.6). We need to show that f is in the image of tr^G_1.

But $\mathrm{tr}^G_1(if') = \mathrm{tr}^G_H \mathrm{tr}^H_1(if') = \mathrm{tr}^G_H(i\,\mathrm{tr}^H_1 f') = \mathrm{tr}^G_H(i)f = f$.

The proof of the second claim is similar. □

Definition 3.2. A chain complex C_* is called:

- *acyclic* if it only has homology in degree 0;
- *weakly induced* if each module is induced, and *weakly induced except in degrees I* if each $C_i, i \notin I$, is induced; and
- *separated* if each $\mathrm{Im}\, d_i \to C_i$ factors through a projective.

Write B_i for $\mathrm{Im}\, d_i \subseteq C_i$. If the inclusion $B_i \to C_i$ factors through a projective then it factors through the injective hull of B_i, call it P_i (injective is equivalent to projective here), and $P_i \to C_i$ is injective since it is so on the socle. Thus we can write $C_i = P_i \oplus C_i'$ and $B_i \subseteq P_i$.

Recall that the Heller translate ΩV of a module V is defined to be the kernel of the projective cover $P_V \to V$ and $\Omega^i V$ for $i \geq 1$ is Ω iterated i times. Similarly $\Omega^{-1}V$ is the cokernel of the injective hull $V \to I_V$ and Ω^{-i} for $i \geq 1$ is its iteration. We let $\Omega^0 V$ denote V with any projective summands removed.

In our context, projective is equivalent to injective, so $\Omega^i\Omega^j V \cong \Omega^{i+j}V$ and if V is induced so is $\Omega^i V$. Also $(\Omega^i V)^* \cong \Omega^{-i}(V^*)$ and $\Omega(V \otimes W) \cong (\Omega V) \otimes W$ modulo projectives. See [3] 2.4, 3.1.6.

Note that $\Omega^2 = 1$ modulo projectives for cyclic p-groups.

Proposition 3.3. *Suppose that the chain complex of G-modules $K_w \to \cdots \to K_0$ is:*

- *acyclic and $H_0(K_*) = L$, say;*
- *weakly induced from H except in at most one degree;*
- *on restriction to H, K_* is separated.*

Then K_* is separated and $L \oplus P \cong K_0 \oplus \Omega^{-1}K_1 \oplus \Omega^{-2}K_2 \oplus \cdots$, for some projective module P.

Proof. We use induction on w, so first consider the case $w = 1$, where we have a short exact sequence $K_1 \to K_0 \to L$.

Now one of K_0, K_1 is induced from H, and $K_1 \to K_0$ factors through a projective over H, so we can apply lemma 3.1 to see that $K_1 \to K_0$ factors through a projective over G. The shifted triangle $K_0 \to L \to \Omega^{-1}K_1$ is stably split, so $L \cong K_0 \oplus \Omega^{-1}K_1$ stably, and thus there are projective modules P and Q such that $L \oplus P \cong K_0 \oplus \Omega^{-1}K_1 \oplus Q$.

We claim that we can take $Q = 0$. Since L is a quotient of K_0, any projective summand of L is also isomorphic to a summand of K_0, and we can cancel, so we can assume that L has no projective summands. But then Q must be isomorphic to a summand of P and we can cancel.

Now for $w \geq 2$ we break up our complex into two: $K_w \to K_{w-1}$, which we shift so that K_{w-1} is in degree 0 and the homology there is $\operatorname{Im} d_{w-1}$, and $\operatorname{Im} d_{w-1} \to K_{w-2} \to \cdots \to K_0$.

These both satisfy the hypotheses of the proposition; in particularif a K_i with $i \leq w-2$ is not induced then both K_w and K_{w-1} are induced, hence so is $\operatorname{Im} d_{w-1}$ by the case $w = 1$.

Thus both complexes are separated, hence so is K_*. Also $L \oplus P' \cong K_0 \oplus \Omega^{-1}K_1 \oplus \cdots \oplus \Omega^{w-2}K_{w-2} \oplus \Omega^{w-1} \operatorname{Im} d_{w-1}$ and $\operatorname{Im} d_{w-1} \oplus P'' \cong K_{w-1} \oplus \Omega^{-1}K_w$, so the last claim follows. □

Definition 3.4. For any pair (V, W) of G-modules we have a Koszul complex $K(V, W)$:

$$\cdots \to \Lambda^2(W) \otimes S^{r-2}(V) \to W \otimes S^{r-1}(V) \to S^r(V),$$

the boundary map is $d(w_1 \wedge \cdots \wedge w_j \otimes s) = \sum_{i=1}^{j} (-1)^{j-i} w_1 \wedge \cdots \wedge \hat{w}_i \wedge \cdots \wedge w_j \otimes w_i s$, $w_i \in W$, $s \in S^*(V)$.

Lemma 3.5. *$K(V, W)$ is exact except at $S^*(V)$, where its homology is $S^*(V/W)$.*

Proof. Ignoring the group action, $K(V, W) \cong K(W, W) \otimes S^*(V/W)$;now $K(W, W)$ is a standard Koszul complex, so known to be exact except in degree 0, where its homology is k. □

Lemma 3.6. *([1]) $K(V, V)$ is split exact in degrees r not divisible by p.*

Proof. The splitting is given by $e(\lambda \otimes v_1 \cdots v_j) = r^{-1} \sum_{i=1}^{j} \lambda \wedge v_i \otimes v_1 \cdots \hat{v}_i \cdots v_j$, $\lambda \in \Lambda^*(W)$, $v_i \in V$. □

Remark. This can be used (inductively on r) to obtain a formula for $S^r(V)$ in terms of $\Lambda^*(V)$ for $r < p$. For cyclic groups of order p, this fact and periodicity are all that is needed to obtain the reduction for all r (see [1]).

Lemma 3.7. *For G cyclic of prime power order and $H < G$ of index p, $S^r(V_{pu})$ is induced in degrees not divisible by p, and $S^{ps}(V_{pu})$ is the tensor induced module $(\uparrow_H^{\otimes, G} S^s(V_u))$, modulo induced modules.*

Proof. Note that $V_{pu} \cong \operatorname{Ind}_H^G V_u$. The first part follows from the lemma above. In general use $S^*(V_u \oplus t_2 V_u \oplus \cdots \oplus t_p V_u) \cong S^*(V_u) \otimes t_2 S^*(V_u) \otimes \cdots \otimes t_p S^*(V_u)$, where $\{1, t_2, \ldots, t_p\}$ is a set of coset representatives for G/H. □

The next lemma is clear.

Lemma 3.8. $K((V_1, W_1) \oplus (V_2, W_2)) \cong K(V_1, W_1) \otimes K(V_2, W_2)$.

Lemma 3.9. *If C^1_* and C^2_* are separated then so is $C^1_* \otimes C^2_*$.*

Proof. We omit sub- and superscripts. We have $B \subseteq P \subseteq C$ (in the notation established after 3.2). The boundary in $C\otimes C$ has image in $(B\otimes C)+(C\otimes B) \subseteq (P\otimes C)+(C\otimes P)$. The $+$ looks unpromising, but we have a left exact sequence $0 \to P \otimes P \to (P \otimes C) \oplus (C \otimes P) \to C \otimes C$. This must split, since everything with a P in it is projective, hence injective, so the image on the right is projective. □

Theorem 3.10. *Let G be a cyclic group of order p^n. For any $0 \le t \le p^n - p^{n-1}$, $K(V_{p^n}, V_t)$ is separated.*

Proof. Use induction on n. First consider $n = 1$. Because $S^r(V_p)$ isprojective except in degrees divisible by p, and because we do not allow $t = p$, all but one of the terms in K are projective, and this forces separability.

For general n, write $S = k[a] \otimes T$, as in 1.1, and let K' be the subcomplex of K defined using T instead of S. This is consistent with the boundary morphisms, because the x used in the definition of T is not in the submodule V_t (this is why we need an upper bound on t). Thus $K = k[a] \otimes K'$, and we need only consider K'.

From 3.7 we know that $S^r(V_{p^n})$ is induced except when$r$ is divisible by p^n, so T^r is induced except when $r = 0$. But for $r = 0$ the corresponding term in K' is either 0 or the last one in that degree, so (graded module)-degree-wise K' is weakly induced except in the highest (complex)-degree.

Now restrict to H of order p^{n-1}. By 2.3, (V_{p^n}, V_t) will decompose as $\oplus_{i=1}^p (V_{p^{n-1}}, W_i)$ with $\dim W_i \le p^{n-1} - p^{n-2}$, and $\operatorname{Res}^G_H K(V_{p^n}, V_t) \cong \otimes_i K(V_{p^{n-1}}, W_i)$, by 3.8.

By induction, as a complex for H, each $K(V_{p^{n-1}}, W_i)$ is acyclic and separated, hence so is $K(V_{p^n}, V_t)$, by 3.9, and thus $K'(V_{p^n}, V_t)$. Now apply 3.3. □

Corollary 3.11. *For $G \cong C_{p^n}$, $r < p^n$ and $p^{n-1} \le t \le p^n$ we have $S^r(V_t) \cong \Omega^{-r}\Lambda^r(V_{p^n-t})$ modulo induced modules.*

In particular, if $r + t > p^n$ then $S^r(V_t)$ is induced.

Proof. $S^*(V_t)$ is the homology of $K(V_{p^n}, V_{p^n-t})$. Use Theorem 3.10 and the last part of Proposition 3.3. As in the proof above, the only possible non-induced term in (module-) degree r is $\Lambda^r(V_{p^n-t}) \otimes S^0(V_{p^n})$. □

We adopt the convention that $S^r = 0$ for $r < 0$.

Corollary 3.12. *With the same conditions as above, $S^r(V_t) \cong \Omega^{p^n-t}S^{p^n-t-r}(V_t)$ modulo induced modules.*

Proof. $S^r(V_t) \cong \Omega^{-r}\Lambda^r(V_{p^n-t}) \cong \Omega^{-r}\Lambda^{p^n-t-r}(V_{p^n-t}) \cong \Omega^{p^n-t-2r}S^{p^n-t-r}(V_t)$ modulo induced modules. □

Remark. It is thus only necessary to calculate $S^r(V_t)$ for $r \le (p^n - t)/2$: the rest will follow.

Corollary 3.13. *Suppose that G has normal Sylow p-subgroup $H \cong C_{p^n}$ and that V is a kG-module such that $\operatorname{Res}^G_H V \cong V_t$. If $r < p^n$ and $p^{n-1} \le t \le p^n$ then $S^r(V) \cong \Omega^{-r}\Lambda^r(\Omega V) \cong \Omega^{t-p^n}(S^{p^n-t-r}(V)^*) \otimes \det(\Omega V)$ modulo summands projective relative to proper subgroups of H (where $*$ denotes the contragredient).*

Proof. As in 1.2, H has complement F, $V/\operatorname{rad} V = L$ and the projective cover of V is $\operatorname{Ind}_F^G L$.

We consider the Koszul complex $K(\operatorname{Ind}_F^G L, \Omega V)$. On restriction to H this becomes $K(V_{p^n}, V_{p^n-t})$. The property of a map factoring through a projective depends only on the restriction to H, so we obtain the same conclusion as in 3.11, which is the first isomorphism. The second follows as in 3.12, but being more careful about the duality. Modulo induced modules, $\Lambda^r(\Omega V) \cong \operatorname{Hom}(\Lambda^{p^n-t-r}(\Omega V), \Lambda^{p^n-t}(\Omega V)) \cong \Lambda^{p^n-t-r}(\Omega V)^* \otimes \det(\Omega V)$, and $\Lambda^{p^n-t-r}(\Omega V)^* \cong (\Omega^{p^n-t-r} S^{p^n-t-r}(V))^* \cong \Omega^{r+t-p^n}(S^{p^n-t-r}(V)^*)$. □

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