MATH39032
Mathematical Modelling of Finance
Worksheet 9

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In the following question tick **all** options that apply.

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1. The interest rate is given by

$$r(t) = \frac{1}{10 + 2t}$$

What is the price of a bond at time $t = 0$, that pays a single coupon amount £2 at $t = 1$, and the face value £5 at $t = 2$?

- ○ 6.05151
- ○ 5.91608
- ○ 5.2381

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2. Which of the following is a valid trading strategy to eliminate risk in the interest rate r?

- ○ $$\Pi = V_1 - \Delta V_2,$$
  where V_1 and V_2 are government bonds with different maturities and different coupon rates.
- ○ $$\Pi = V - \Delta r,$$
  where r is the interest rate.
- ○ $$\Pi = V_1 - \Delta V_2,$$
  where V_1 and V_2 are bonds trading in different countries.
- ○ $$\Pi = V_1 - \Delta V_2,$$
  where V_1 and V_2 are zero coupon government bonds with different maturities.

For both questions on this page, assume that we price bonds under the Vasicek model with a given solution

$$V = Ze^{A(t;T)-rB(t;T)}$$

where

$$A = \frac{1}{\gamma^2}(B - T + t)(\eta\gamma - \tfrac{1}{2}\beta - \lambda\gamma\beta^{\frac{1}{2}}) - \frac{\beta B^2}{4\gamma}$$

and

$$B = \frac{1}{\gamma}(1 - e^{-\gamma(T-t)}).$$

3. Consider a bond close to maturity, with $T - t = \epsilon$ and ϵ is a small number. Derive an approximation to the bond in terms of ϵ (ignoring terms $O(\epsilon^2)$).

   ◯ $$V \approx Z \exp\left[\frac{\nu - 1/2\beta}{\gamma} - \beta\gamma - r\gamma\epsilon\right]$$

   ◯ $$V \approx Ze^{-r\epsilon}$$

   ◯ $$V \approx Z \exp\left[2\epsilon\frac{\eta\gamma - \frac{1}{2}\beta - \lambda\gamma\beta^{\frac{1}{2}}}{\gamma} - r\epsilon\right]$$

   ◯ $$V \approx Z \exp\left[\frac{\epsilon\nu\gamma}{\gamma^2} - \beta\epsilon - r\gamma\epsilon\right]$$

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4. There exists a call option $C(r,t)$ with strike price X and maturity T, that give the holder the option to buy a **bond** with maturity $T_1 > T$ and face value Z (so in this case a bond is the underlying asset). The option price satisfies the Vasicek PDE, but what are the boundary conditions?

   ◯ $$C(r,T) = \max(Ze^{A(t;T)-rB(t;T)} - X, 0)\ , \qquad C \to 0 \text{ as } r \to \infty$$

   ◯ $$C(r,T) = \max(Ze^{A(T;T_1)-rB(T;T_1)} - X, 0)\ , \qquad C \to 0 \text{ as } r \to \infty$$

   ◯ $$C(r,T) = \max(Ze^{A(t;T)-rB(t;T)} - X, 0)\ , \qquad C \to X \text{ as } r \to \infty$$

   ◯ $$C(r,T) = \max(Ze^{A(T;T_1)-rB(T;T_1)} - X, 0)\ , \qquad C \to Ze^{A(t;T_1)-rB(t;T_1)} - Xe^{A(t;T)-rB(t;T)} \text{ as } r \to \infty$$
