MATH39032
Mathematical Modelling of Finance
Worksheet 7

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In the following question tick **all** options that apply.

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1. Consider the Black-Scholes PDE with dividends,

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + (r - D)S\frac{\partial V}{\partial S} - rV = 0.$$

Derive the boundary conditions at large and small S for a call option with terminal condition

$$V(S,T) = \max(S - X, 0).$$

- ○ $V \to 0$ as $S \to 0$ and $V \to Se^{-D(T-t)} - Xe^{-r(T-t)}$ as $S \to \infty$.
- ○ $V \to 0$ as $S \to 0$ and $V \to S - Xe^{-r(T-t)}$ as $S \to \infty$
- ○ $V \to 0$ as $S \to 0$ and $V \to S$ as $S \to \infty$
- ○ $V \to 0$ as $S \to 0$ and $V \to Se^{-D(T-t)}$ as $S \to \infty$
- ○ $V \to e^{-D(T-t)}$ as $S \to 0$ and $V \to Se^{-D(T-t)}$ as $S \to \infty$

Apply the transformation $S = Xe^{x+D(T-t)}$, $V = Xv$ and $t = T - \frac{1}{\frac{1}{2}\sigma^2}\tau$ and the resulting PDE is:-

○
$$-\frac{\partial v}{\partial \tau} + \frac{\partial^2 v}{\partial x^2} + (\rho - \nu - 1)\frac{\partial V}{\partial x} - \rho v = 0.$$

○
$$-\frac{\partial v}{\partial \tau} + \frac{\partial^2 v}{\partial x^2} + (\rho - 1)\frac{\partial V}{\partial x} - \rho v = 0.$$

○
$$\frac{\partial v}{\partial \tau} = \frac{\partial^2 v}{\partial x^2} + (\rho - \nu - 1)\frac{\partial V}{\partial x} - (\rho - \nu)v.$$

where $\rho = \frac{2r}{\sigma^2}$ and $\nu = \frac{2D}{\sigma^2}$. The initial condition is $v(x, \tau = 0) = max(e^x - 1, 0)$. The boundary conditions are now:-

- ○ $v \to 0$ as $x \to -\infty$ and $v \to e^{x+\nu\tau} - e^{\rho\tau}$ as $x \to \infty$
- ○ $v \to 0$ as $x \to -\infty$ and $v \to e^x - 1$ as $x \to \infty$
- ○ $v \to 0$ as $x \to -\infty$ and $v \to e^x - e^{-\rho\tau}$ as $x \to \infty$

2. Calculate the value of a call on an asset (value S) that pays out a dividend $d_1 S(t_{d_1}^-) = 0.1S(t_{d_1}^-)$ at time ($t_{d_1} = 0.5$). The strike price is $X = 90$, $S_0 = 87.2$, $r = 0.02$, $\sigma = 0.3$ and $T = 1$.

- ◯ 6.45
- ◯ 5.80
- ◯ 1.72
- ◯ 9.95

3. Calculate the value of a call on an asset (value S) that pays out a dividend $d_1 S(t_{d_1}^-) = 0.1S(t_{d_1}^-)$ at time ($t_{d_1} = 0.25$). The strike price is $X = 90$, $S_0 = 87.2$, $r = 0.02$, $\sigma = 0.3$ and $T = 1$. What is the difference in value from question 2?

- ◯ Value is lower
- ◯ No difference
- ◯ Value is higher

4. Calculate the value of a put option on an asset (value S) that pays out a dividend $d_1 S(t_{d_1}^-) = 0.1S(t_{d_1}^-)$ at time ($t_{d_1} = 0.5$). The strike price is $X = 90$, $S_0 = 87.2$, $r = 0.02$, $\sigma = 0.3$ and $T = 1$.

- ◯ 1.09
- ◯ 1.72
- ◯ 1.55

5. Calculate the value of a put option on an asset that pays out a **continuous** dividend yield $D = 0.1$. The strike price is $X = 90$, $S_0 = 87.2$, $r = 0.02$, $\sigma = 0.3$ and $T = 1$.

- ◯ 1.53
- ◯ 1.14
- ◯ 1.67

6. What is the difference between the values in the continuous and discrete dividend cases?

- ◯ Value is lower
- ◯ Almost no difference
- ◯ Value is higher