MATH39032
Mathematical Modelling of Finance
Worksheet 4

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In the following question tick **all** options that apply.

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1. Consider a financial contract $V(S_1, S_2, t)$ with the payoff

$$V(S_1, S_2, T) = \max(S_1 - X_1, S_2 - X_2, 0).$$

Assuming the standard Black-Scholes model holds, what is the appropriate boundary condition for $S_1 \to 0$ and $S_2 \to 0$?

- ◯ $V = -X_1 e^{-r(T-t)}$
- ◯ $V = 0$
- ◯ $V = -X_2 e^{-r(T-t)}$

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2. Consider that the financial contract $V(S, t)$ satisfies the following non-standard PDE

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + r(S + \theta)\frac{\partial V}{\partial S} - rV = 0.$$

If $V(S, T) = S - X$, find a solution of the form

$$V = A(t)S + B(t)$$

- ◯ $V = Se^{-\theta(T-t)} - Xe^{-r(T-t)}$
- ◯ $V = Se^{-\frac{r\theta}{S}(T-t)} - Xe^{-r(T-t)}$
- ◯ $V = S - Xe^{-r(T-t)} - r\theta e^{-r(T-t)}$
- ◯ $V = S - Xe^{-r(T-t)} + \theta\left[1 - e^{-r(T-t)}\right]$

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3. Consider that the financial contract $V(Z,t)$ satisfies the following non-standard PDE:

$$\frac{\partial V}{\partial t} + aZ^2\frac{\partial^2 V}{\partial Z^2} = 0,$$

with terminal condition

$$V(Z, t = T) = \max(1 - Z, 0)$$

and boundary conditions

$$V \to 0 \quad \text{as } Z \to \infty$$
$$V = 1 \quad \text{at } Z = 0.$$

Using the Black-Scholes formulae from the notes find the value of the contract for $Z = 0.95$, $t = 0$ when $a = 0.02$ and $T = 0.75$. You may calculate $N(x)$ using the tables attached to examples 3 or use an online calculator.

- ◯ 0.0452
- ◯ 0.0952
- ◯ 0.417
- ◯ -0.582

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4. Consider that the financial contract $V(S,t)$ satisfies the following BS PDE

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0.$$

Seeking a solution by separation of variables

$$V(S,t) = A(t)B(S),$$

you obtain:

- ◯ $A(t) = e^{-\lambda(T-t)}$ and $B(S) = S^\alpha$ where α is the root of an equation involving λ.
- ◯ $A(t) = e^{-r(T-t)}$ and $B(S) = S^\alpha$ where $\alpha = 1$ or $\alpha = -2r/\sigma^2$.
- ◯ Nothing – can't be solved.
