MATH39032
Mathematical Modelling of Finance
Worksheet 3

Dr P. V. Johnson

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In the following question tick **all** options that apply.

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1. Consider the mean-reverting process as given by

$$dX = \kappa(\theta - X)dt + \sigma dW.$$

Given the process

$$Y = f(X,t) = e^{\kappa t}X,$$

use Itô's lemma to find an expression for dY.

- ◯ $$df = e^{\kappa t}\kappa(\theta - X)dt + \sigma e^{\kappa t}dW$$
- ◯ $$df = \kappa\theta e^{\kappa t}dt + \sigma e^{\kappa t}dW$$
- ◯ $$df = \left[\kappa\theta + \kappa(1 - X)e^{\kappa t}\right]dt + \sigma e^{\kappa t}dW$$

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2. We wish to derive the price of a financial contract $V(S_1, S_2, t)$ using Black-Scholes analysis, how should we set up our hedging portfolio?

- ◯ $$\Pi = V - \Delta_1 S_1 + \Delta_2 S_2$$
- ◯ $$\Pi = V - \Delta S_1 - \Delta S_2$$
- ◯ $$\Pi = V - \Delta_1 S_1 - \Delta_2 S_2 - \Delta_{12} S_1 S_2$$

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3. We wish to derive the price of a financial contract $V(S,t)$ using Black-Scholes analysis in which the contract pays the holder $qSdt$ shares at each point in time for a fixed constant q.

   If the hedging portfolio is $\Pi = V - \Delta S$ and the share does not pay any dividends, what is the change in portfolio value $d\Pi$?

   ○ $$d\Pi = dV - \Delta dS - q\Delta S dt$$

   ○ $$d\Pi = dV - \Delta dS + q\Delta S dt$$

   ○ $$d\Pi = dV + qSdt - \Delta dS$$

   ○ $$d\Pi = dV - qSdt - \Delta dS$$

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4. Consider a financial contract $V(S_1, S_2, t)$ with the payoff

$$V(S_1, S_2, T) = \max(S_1 - X_1, S_2 - X_2, 0)$$

   If at $t = T$, the value of the second share $S_2(T) = 2X_2$, which of the following correctly describes the payoff at maturity

   ○ $$V(S_1, S_2 = 2X_2, T) = \begin{cases} S_1 - X_1 & \text{if } S_1 \geq X_1 \\ 0 & \text{if } S_1 < X_1 \end{cases}$$

   ○ $$V(S_1, S_2 = 2X_2, T) = \begin{cases} S_1 - X_2 & \text{if } S_1 \geq X_2 \\ X_2 & \text{if } S_1 < X_2 \end{cases}$$

   ○ $$V(S_1, S_2 = 2X_2, T) = \begin{cases} S_1 - X_1 & \text{if } S_1 \geq X_1 + X + 2 \\ X_2 & \text{if } S_1 < X_1 + X_2 \end{cases}$$

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5. Consider an options contract defined as the best of two assets non- dividend paying assets, S_1 and S_2, that are correlated Geometric Brownian motions. Derive the PDE pricing equation using hedging arguments.

   ○ $$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma_1^2 S_1^2 \frac{\partial^2 V}{\partial S_1^2} + \frac{1}{2}\sigma_2^2 S_2^2 \frac{\partial^2 V}{\partial S_2^2} + \frac{1}{2}\sigma_1\sigma_1 S_1 S_2 \frac{\partial^2 V}{\partial S_1 \partial S_2} + rS_1 \frac{\partial V}{\partial S_1} + rS_2 \frac{\partial V}{\partial S_2} - rV = 0$$

   ○ $$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma_1^2 S_1^2 \frac{\partial^2 V}{\partial S_1^2} + \frac{1}{2}\sigma_2^2 S_2^2 \frac{\partial^2 V}{\partial S_2^2} + \rho_{12}\sigma_1\sigma_1 S_1 S_2 \frac{\partial^2 V}{\partial S_1 \partial S_2} + rS_1 \frac{\partial V}{\partial S_1} + rS_2 \frac{\partial V}{\partial S_2} - rV = 0$$

   ○ $$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma_1^2 S_1^2 \frac{\partial^2 V}{\partial S_1^2} + \frac{1}{2}\sigma_2^2 S_2^2 \frac{\partial^2 V}{\partial S_2^2} + r(S_1 + S_2) \frac{\partial^2 V}{\partial S_1 \partial S_2} - rV = 0$$
