MATH39032
Mathematical Modelling of Finance
Worksheet 2

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In the following question tick **all** options that apply.

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1. If $dS = \kappa(\theta - S)dt + \sigma S dW$ and A and n are constants then find the stochastic differential equations satisfied by $f(S,t) = Ae^{-\kappa t}S^n$.

   ○
   $$df = \left(\frac{\theta}{S} + \frac{1}{2}n(n-1)\sigma^2 \right) S dt + n\sigma S dW$$

   ○
   $$df = \left(-\kappa + n\kappa \left[\frac{\theta}{S} - 1 \right] + \frac{1}{2}n(n-1)\sigma^2 \right) f dt + n\sigma f dW$$

   ○
   $$df = \left(\frac{\theta}{S} + \frac{1}{2}n(n-1)\sigma^2 \right) f dt + n\sigma f dW$$

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2. Consider two uncorrelated assets ($\rho_{ij} = 0$ if $i \neq j$),

$$dS_1 = \mu_1 S_1 dt + \sigma_1 S_1 dW_1$$

and

$$dS_2 = \mu_2 S_2 dt + \sigma_2 S_2 dW_2.$$

Find the stochastic differential equations satisfied by $f(S_1, S_2) = \log(S_1 S_2)$.

   ○
   $$df = \left(\mu_1 + \mu_2 - \frac{1}{2}\sigma_1^2 - \frac{1}{2}\sigma_2^2 \right) dt + \sigma_1 dW_1 + \sigma_2 dW_2$$

   ○
   $$df = (\mu_1 + \mu_2) f dt + \sigma_1 f dW_1 + \sigma_2 f dW_2$$

   ○
   $$df = \left(\mu_1 S_2 + \mu_2 S_1 - \frac{1}{2}\sigma_1^2 S_2^2 - \frac{1}{2}\sigma_2^2 S_1^2 \right) dt + \sigma_1 \frac{S_2}{S_1} dW_1 + \sigma_2 \frac{S_1}{S_2} dW_2$$

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3. Consider two uncorrelated assets ($\rho_{ij} = 0$ if $i \neq j$),

$$dS_1 = \kappa(\phi - S_2)dt + \sigma S_1 dW_1$$

and

$$dS_2 = \phi(\kappa - S_1)dt + \sigma S_2 dW_2.$$

Find the stochastic differential equations satisfied by $f(S_1, S_2) = S_1 S_2$.

○
$$df = [\kappa\phi(S_1 + S_2) - (\phi + \kappa)f]\,dt + \sigma f(dW_1 + dW_2)$$

○
$$df = \kappa\phi\left[(S_1 + S_2) - \left(\frac{1}{\phi} + \frac{1}{\kappa}\right)f\right]dt + \sigma(S_1^2 + S_2^2)(dW_1 + dW_2)$$

○
$$df = \kappa\phi\left[(S_1 + S_2) - \left(\frac{S_2^2}{\phi} + \frac{S_1^2}{\kappa}\right)\right]dt + \sigma f(dW_1 + dW_2)$$

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4. Try the trading games, you can find them under the examples tab on the course website:-
https://mybinder.org/v2/gh/pjohno/MATH39032/master?filepath=notebooks/Simulating Strategies.ipynb
and
https://mybinder.org/v2/gh/pjohno/MATH39032/master?filepath=notebooks/Real Data Strategies.ipynb