MATH39032

Solutions 1

1. (a) On exercising a call option, the holder receives $\max(S-X,0)$. If $S-X$ is very large today then one would expect the option to be worth a lot as it is quite likely that it will have a high value at expiry.

   (b) Similarly, is $S-X$ is a lot less than zero there is little value in holding the option as it is unlikely to be exercised at expiry. In this case the option is worth very little.

   (c) If the underlying asset is very volatile then the option becomes more valuable, as any upside profit is realised but any losses are not. The more volatile an underlying it is the better it is for the holder of the option.

2. (a) She would exercise if $S > \$50$ and would make a profit if $S > \$52.50$. See figure 1 for diagram.

   (b) See figure 2

   (c) See figures 3 and 4.

3. (a) He would exercise if $S < \$60$ and would make a profit if $S < \$56$. See figure 5 for diagram.

   (b) See figure 6

   (c) See figures 7 and 8

4. (a) Short one share (i.e. sell without actually owning it) at the current price S and go long in the forward contract. At the expiry of the forward the arbitrageur's amount S has grown to Se^{rT} and he pays F to take delivery of the share. This share is used to provide the share which he shorted at the start of the contract. This realises a risk-free net gain of

$$Se^{rT} - F > 0$$

   (b) The price drops to $S-D$. The person holding the share will receive an amount D and if the price remained at S then they would have made a risk free profit of D, thus the share price must drop to compensate.

5. Since the holder of one share just before a two-for-one split will hold two just after, and since this is only a marginal change, the stock price must be halved on the introduction of the new shares. The exercise prices of options are therefore also halved, as are their values.

6. (a) Using $B(r,t) = Ae^{-r(T-t)}$, where the bond pays A at maturity ($t = T$). Here, $r = .05$, $T = t = 10$, $A = 100$, and so $B = 100 \times e^{-.05\times 10} = 60.65$.

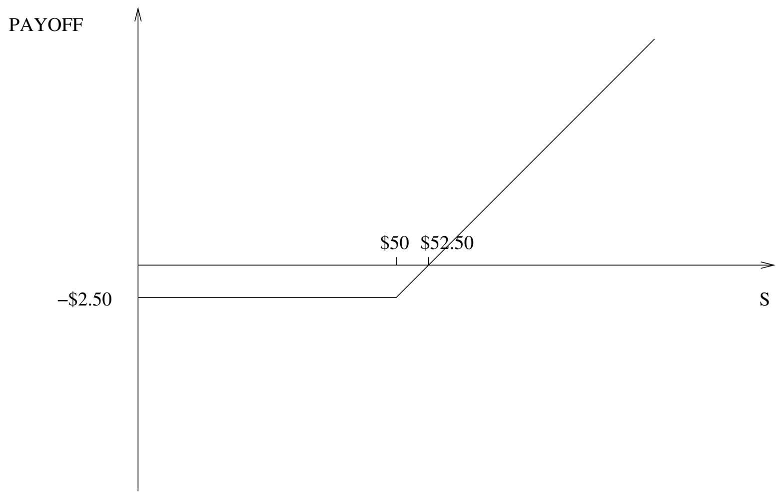

Figure 1: The profit for the holder of a European call option costing \$2.50 and with strike price \$50

(b) First discount the bond value from maturity five years, during which period $r = .06$. This gives a discounted value of

$$1000000e^{-.06\times 5} = 740818$$

This value then has to be discounted back a further five years, to today, at an annual interest rate of $r = .05$, giving

$$740818e^{-.05\times 5} = 576950$$

(c) As stated in the hint, both dividend payments can be treated as mini-bonds, ans so the total value of the bond today is the discounted value of the bond at maturity, together with the discounted values of the two coupons, with everything discounted at an annual rate of $r = .025$, i.e.

$$100e^{-.025\times 3}+5e^{-.025\times 2}+5e^{-.025\times 1} = 92.77+4.76+4.88 = 102.41.$$

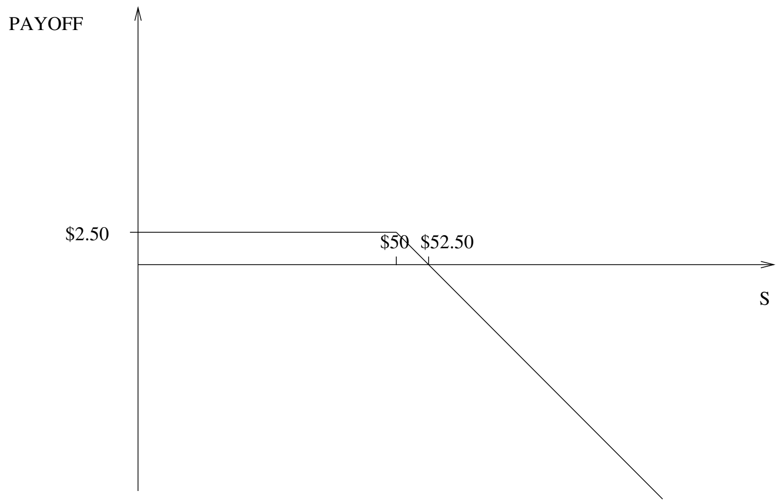

Figure 2: The profit for the writer of a European call option costing $2.50 and with strike price $50

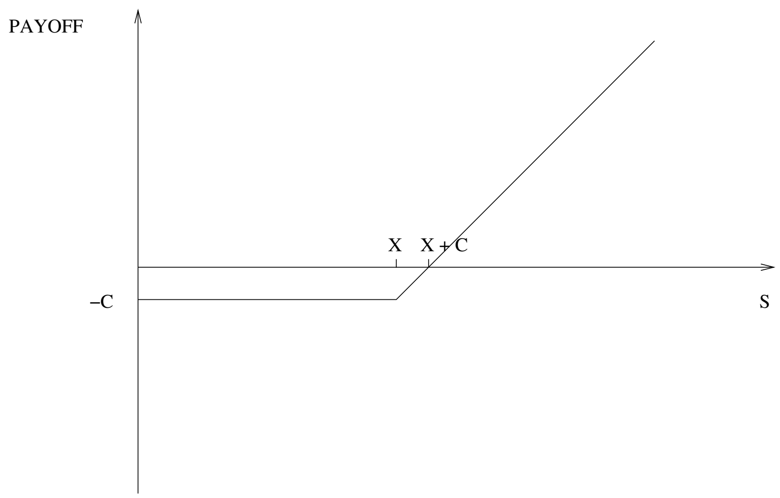

Figure 3: The profit for the holder of a European call option costing C and with strike price X

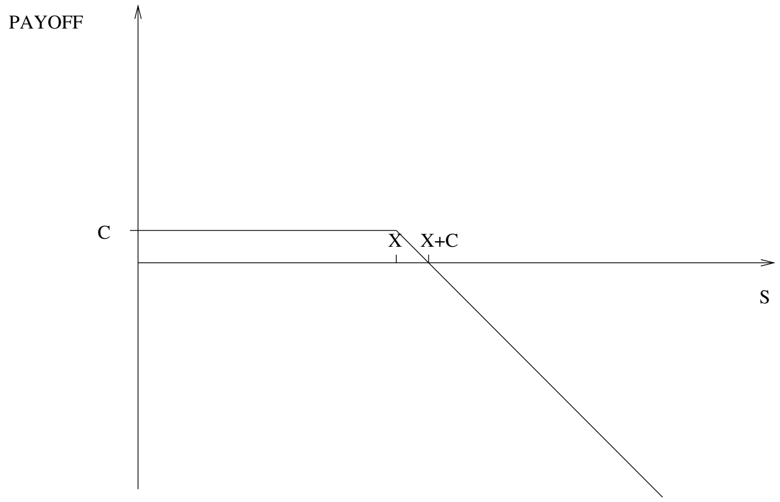

Figure 4: The profit for the writer of a European call option costing C and with strike price X

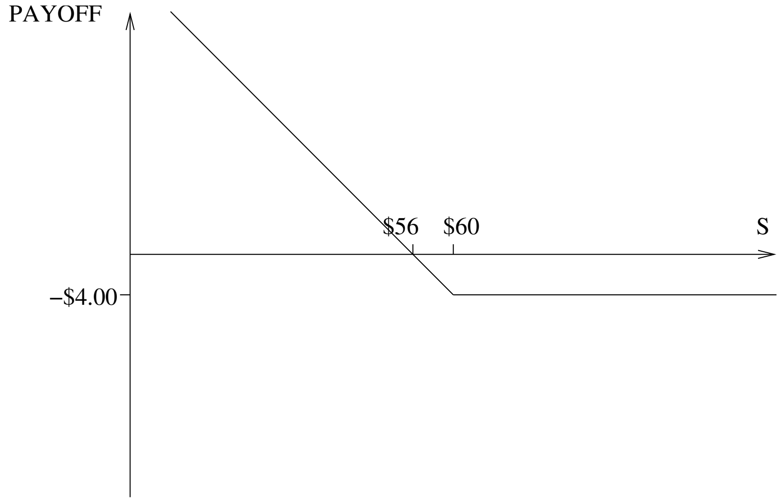

Figure 5: The profit for the holder of a European put option costing $4 and with strike price $60

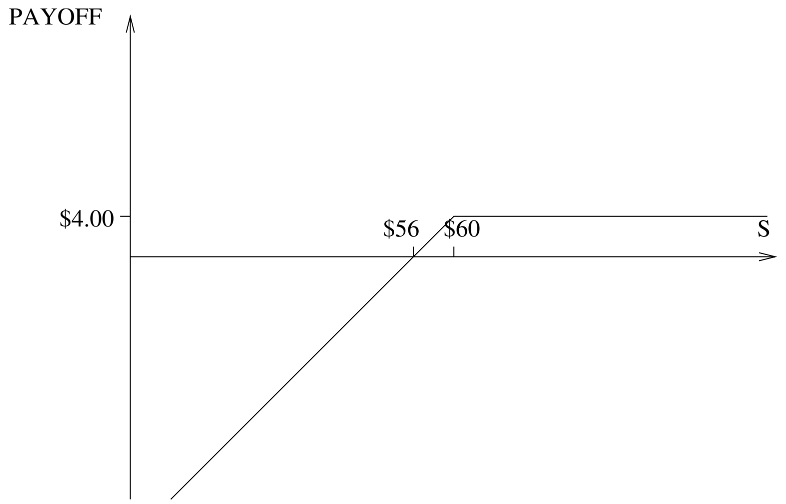

Figure 6: The profit for the writer of a European put option costing $4 and with strike price $60

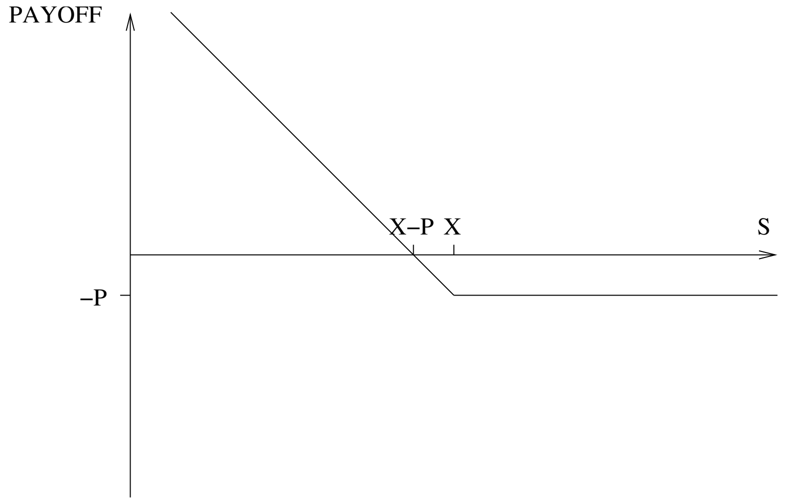

Figure 7: The profit for the holder of a European put option costing P and with strike price X

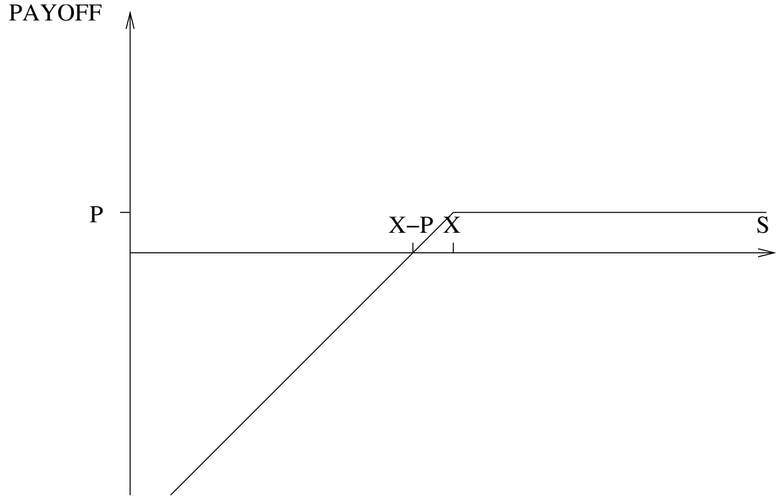

Figure 8: The profit for the writer of a European put option costing P and with strike price X