Password is Math5_1718

2018

A1 If the dividend is £1, there will be a drop in the value of S after 1, 2 and 3 years.

So the fair value after 4 years will be

$$200 \cdot e^{4 \times .05} - 1 \cdot e^{3 \times .05} - 1 \cdot e^{2 \times .05} - 1 e^{.05} \quad \underline{7}$$

$= 244.28 - 1.1618 - 1.105 - 1.051$

$= 240.96$ = F $\quad \underline{3}$

So investor could go short on forward

(1) Borrow £200 at $t=0$ to buy asset

(2) Pay off £1 at $t = 1, 2, 3$

(3) Receive £245 from contract

$\Rightarrow$ profit (risk free) £4.04 $\quad \underline{5}$

seem similar (15)

A2 (i) $V = S^n f(t)$

$V_t = S^n f'(t)$

$V_S = n S^{n-1} f(t)$

$V_{SS} = n(n-1) S^{n-2} f(t)$ $\underline{3}$

Substitute into BSE

$S^n f'(t) + \frac{1}{2}\sigma^2 n(n-1) S^n f(t)$
$+ rn S^n f(t) - r S^n f(t) = 0$

$f' + \left[\frac{1}{2}\sigma^2 n(n-1) + r(n-1)\right] f = 0$

$f = e^{\left[\frac{1}{2}\sigma^2 n(n-1) + r(n-1)\right](T-t)}$ $\underline{8}$

(ii) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

$\Rightarrow \exp(S) = \sum_{n=0}^{\infty} \frac{S^n}{n!}$

$V = \sum_{n=0}^{\infty} \frac{e^{\left[\frac{1}{2}\sigma^2 (n-1)n + r(n-1)\right](T-t)}}{n!}$ $\underline{4}$

15

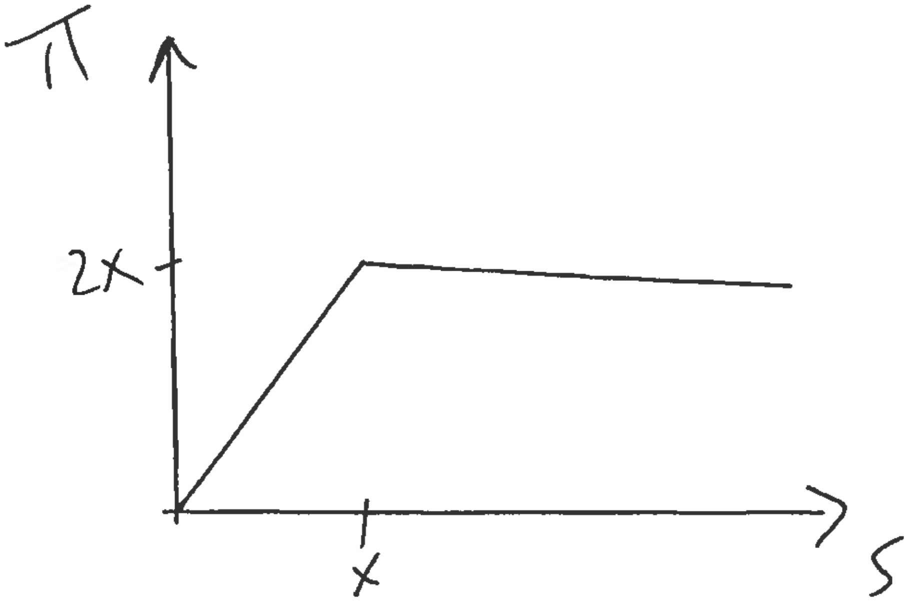
Unseen

A3 (i) $\pi(t=T) = 2S - 2C(X)$ 4

$$= 2S - 2\max(S - X, 0)$$

For $0 \leq S \leq X$, $\pi = 2S$

For $S \geqslant X$, $\pi = 2S - 2(S - X)$
$$= 2X$$



3

(ii) $\pi(t=T) = -S + 3P(X_1) - C(X_2)$

$$= -S + 3\max(X_1 - S, 0) - \max(S - X_2, 0)$$

If $x_1 > x_2$ 12

For $0 < s < x_2$

$$\pi = -s + 3(x_1 - s) = 3x_1 - 4s$$

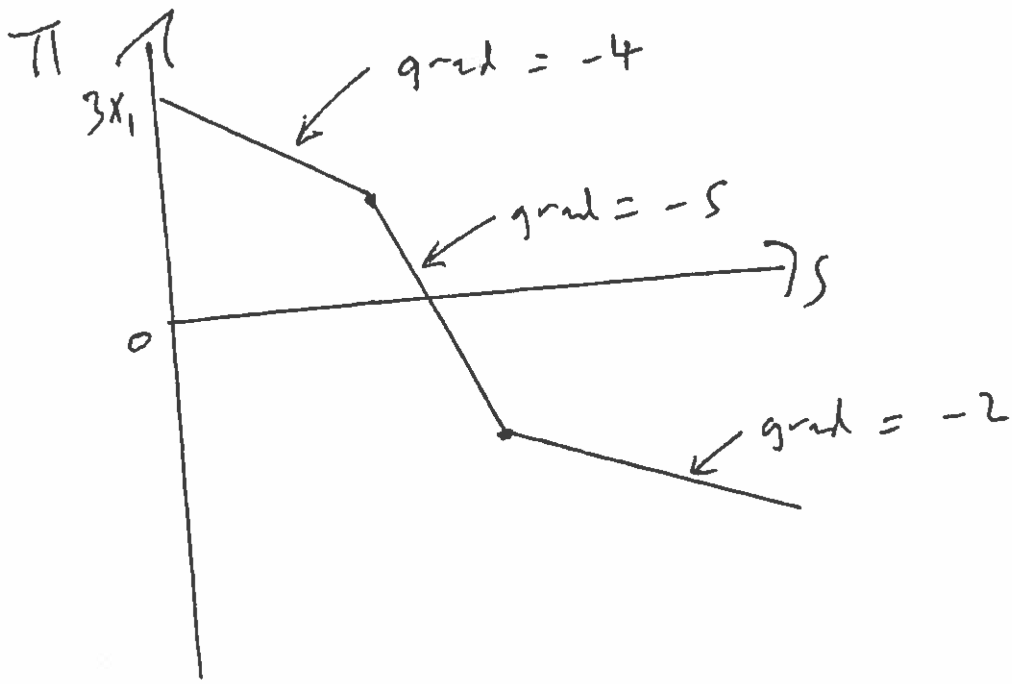
For $x_2 < s < x_1$

$$\pi = -s + 3(x_1 - s) + (x_2 - s)$$

$$= 3x_1 + x_2 - 5s$$

For $s > x_1$

$$\pi = -s + (x_2 - s) = x_2 - 2s$$



4

B

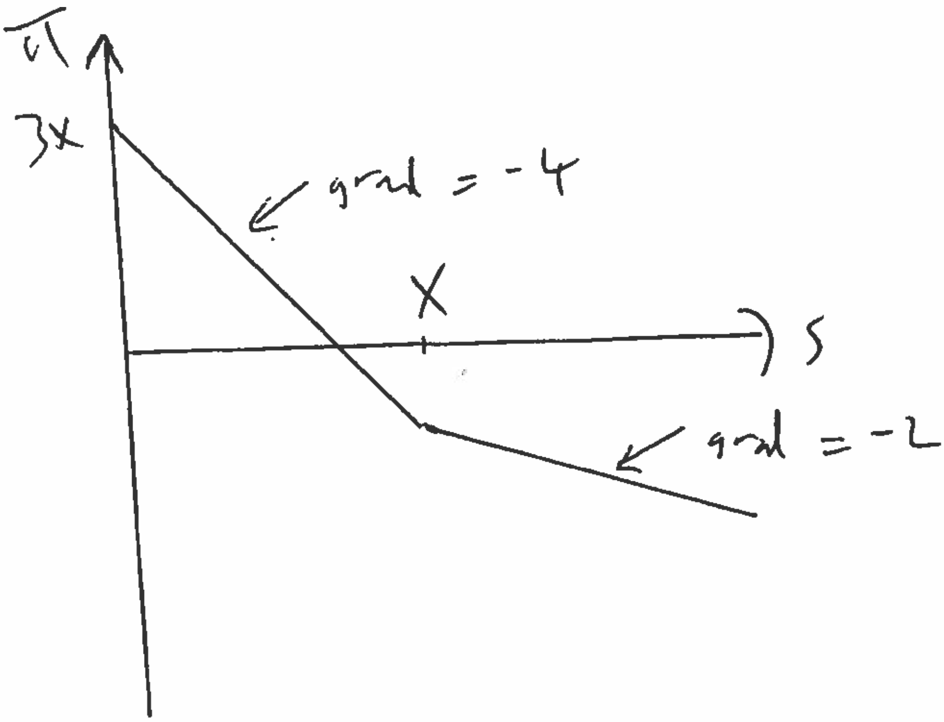
If $X_1 = X_2 = X$

For $0 < S < X$

$\pi = -S + 3(X - S) = 3X - 4S$

For $X < S < \infty$

$\pi = -S + X_2 - S = X_2 - 2S$



4

If $X_1 < X_2$

For $0 < S < X_1$

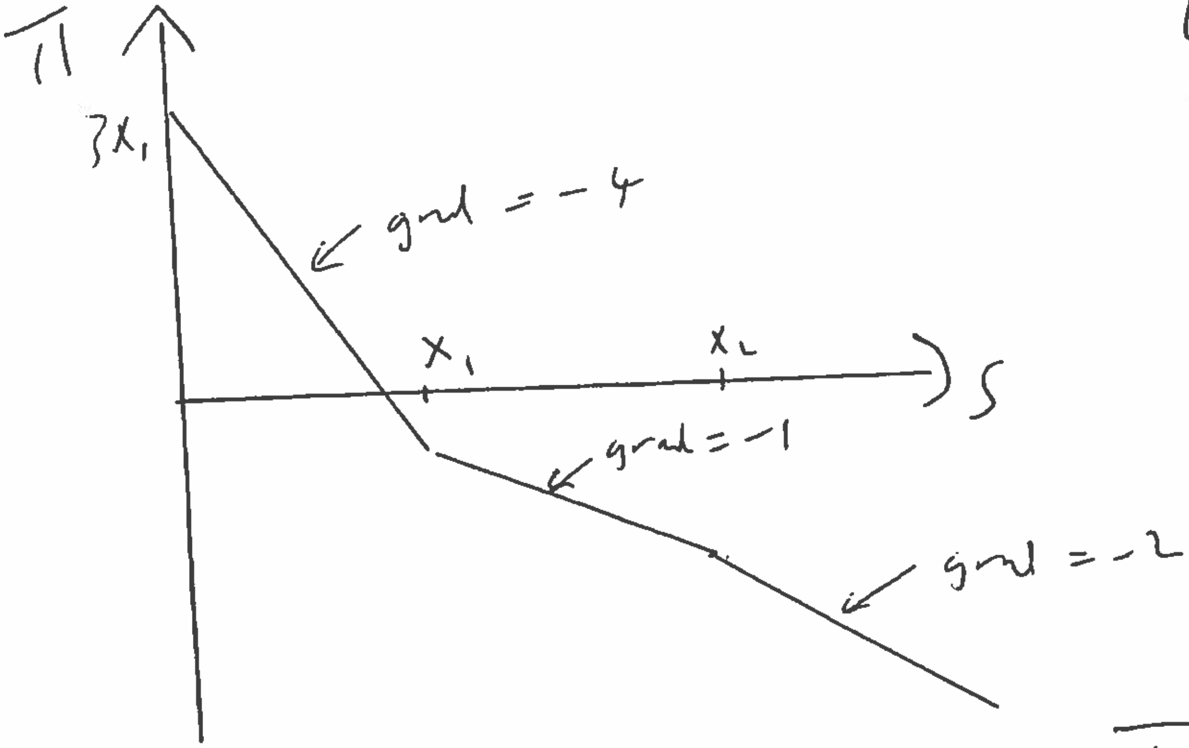
$\pi = -S + 3(X_1 - S) = 3X_1 - 4S$

For $X_1 < S < X_2$

$\pi = -S + 0$

For $S > X_2$

$\pi = -S - (S - X_2) = -2S + X_2$



$\overline{\text{I}}$

see similar

15

A4 (i) $df = \frac{\partial f}{\partial t} dt + \frac{\partial f}{\partial S} dS$ 11

$$+ \frac{1}{2} \frac{\partial^2 f}{\partial S^2} dS^2 + .$$

$$= \frac{\partial f}{\partial t} dt + \frac{\partial f}{\partial S} [A dt + B dw]$$

$$+ \frac{1}{2} \frac{\partial^2 f}{\partial S^2} B^2 dt$$ 3

(ii) If $A = \mu S$, $B = \sigma S^{\beta}$

$$df = \left[\mu S \frac{\partial f}{\partial S} + \frac{\partial f}{\partial t} + \frac{1}{2} \sigma^2 S^{2\beta} \frac{\partial^2 f}{\partial S^2} \right] dt$$

$$+ \sigma S^{\beta} \frac{\partial f}{\partial S} dw$$ 3

(iii) If $\Pi = V - \Delta S$

$$d\Pi = dV - \Delta dS$$

$$= \left[\mu S \frac{\partial V}{\partial S} + \frac{\partial V}{\partial t} + \frac{1}{2} \sigma^2 S^{2\beta} \frac{\partial^2 V}{\partial S^2} \right] dt$$

$$+ \sigma S^{\beta} \frac{\partial V}{\partial S} dw$$

$$- \Delta [\mu S dt + \sigma S^{\beta} dw]$$ 4

$$\frac{2}{2}$$

If $\Delta = \frac{\partial V}{\partial S}$

$$d\Pi = \left[\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^{2\beta}\frac{\partial^2 V}{\partial S^2}\right]dt$$

no arbitrage

$$= r\,dt\left[V - \frac{\partial V}{\partial S}S\right]$$

$$\Rightarrow \frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^{2\beta}\frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0$$

$$\frac{}{3}$$

Ito lemma seen in lectures/ex sheets (15)
CEV process unseen